# OSCILLATIONS

1. A small sphere carrying a charge q is hanging in between two parallel plates by a string of length L. Time period of pendulum is T_0. When parallel plates are charged, the time period changes to T. The ratio T/T_0 is equal to
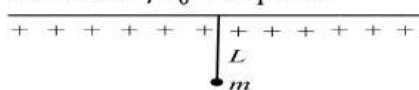
   a) $\left(\dfrac{g+\frac{qE}{m}}{g}\right)^{1/2}$
   b) $\left(\dfrac{g}{g+\frac{qE}{m}}\right)^{3/2}$
   c) $\left(\dfrac{g}{g+\frac{qE}{m}}\right)^{1/2}$
   d) None of these

2. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3 \times 1000 \text{ kg} - \text{m}^3)$. What relationship between t and t_0 is true?
   a) $t = t_0$
   b) $t = t_0/2$
   c) $t = 2t_0$
   d) $t = 4t_0$

3. As a body performs S.H.M., its potential energy U. Varies with time as indicated in
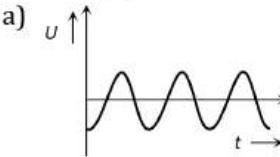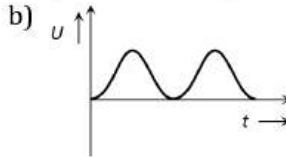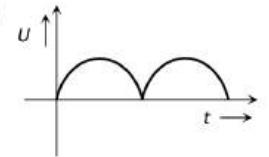
   a)
   b)
   c)
   d)
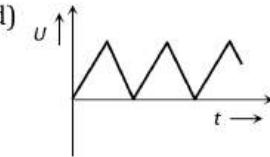
4. Two simple pendulum of length 0.5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed… oscillations.
   a) 5
   b) 1
   c) 2
   d) 3

5. A simple harmonic oscillator has a period of $0.01\ s$ and an amplitude of $0.2\ m$. The magnitude of the velocity in $m\sec^{-1}$ at the centre of oscillation is
   a) 20π
   b) 100
   c) 40π
   d) 100π

6. A body has a time period T_1 under the action of one force and T_2 under the action of another force, the square of the time period when both the forces are acting in the same direction is
   a) $T_1^2 T_2^2$
   b) T_1^2/T_2^2
   c) $T_1^2 + T_2^2$
   d) $T_1^2 T_2^2/(T_1^2 + T_2^2)$

7. For a simple pendulum the graph between L and T will be
   a) Hyperbola
   b) Parabola
   c) A curved line
   d) A straight line

8. A mass of 4 kg suspended from a spring of force constant 800 Nm^{-1} executes simple harmonic oscillations. If the total energy of the oscillator is $4J$, the maximum acceleration (in ms^{-2}) of the mass is
   a) 5
   b) 15
   c) 45
   d) 20

9. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of
   a) $(2/3)k$
   b) $(3/2)k$
   c) $3k$
   d) $6k$

10. There is a body having mass m and performing S.H.M. with amplitude a. There is a restoring force $F = -Kx$, where x is the displacement. The total energy of body depends upon
   a) K, x b) K, a c) K, a, x d) K, a, v

11. If a body of mass $0.98\ kg$ is made to oscillate on a spring of force constant $4.84\ N/m$, the angular frequency of the body is
   a) $1.22\ rad/s$ b) $2.22\ rad/s$ c) $3.22\ rad/s$ d) $4.22\ rad/s$

12. The amplitude of vibration of a particle is given by $a_m = (a_0)/(a\omega^2 - b\omega + c)$; where a_0, a, b and c are positive. The condition for a single resonant frequency is
   a) $b^2 = 4ac$ b) $b^2 > 4ac$ c) $b^2 = 5ac$ d) $b^2 = 7ac$

13. The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is
   a) Greater than T b) Less than T c) Equal to T d) Cannot be compared

14. In a simple pendulum, the period of oscillation T is related to length of the pendulum l as
   a) $\dfrac{l}{T} = $ constant   b) $\dfrac{l^2}{T} = $ constant   c) $\dfrac{l}{T^2} - $ constant   d) $\dfrac{l^2}{T^2} = $ constant

15. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?
   a) $\dfrac{1}{6}$ s   b) $\dfrac{1}{4}$ s   c) $\dfrac{1}{3}$ s   d) $\dfrac{1}{12}$ s

16. A mass m is suspended from a spring of length l and force constant K. The frequency of vibration of the mass is f_1. The spring is cut into two equal parts and the same mass is suspended from one of the parts. The new frequency of vibration of mass is f_2. Which of the following relations between the frequencies is correct
   a) $f_1 = \sqrt{2}f_2$ b) $f_1 = f_2$ c) $f_1 = 2f_2$ d) $f_2 = \sqrt{2}f_1$

17. How does time period of a pendulum very with length?
   a) $\sqrt{l}$ b) $\sqrt{\dfrac{l}{2}}$ c) $\dfrac{1}{\sqrt{l}}$ d) $2l$

18. A particle is vibrating in a simple harmonic motion with an amplitude of $4\ cm$. At what displacement from the equilibrium position, is its energy half potential and half kinetic
   a) $1\ cm$ b) $\sqrt{2}\ cm$ c) $3\ cm$ d) $2\sqrt{2}\ cm$

19. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height $2R$ above the earth's surface where R is the radius of the earth. The value of (T_1/T_2) is
   a) $1/9$ b) $1/3$ c) $\sqrt{3}$ d) 3

20. A ball of mass $(m)0.5\ kg$ is attached to the end of a string having length $(L)0.5\ m$. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is $324\ N$. The maximum possible value of angular velocity of ball (in radian/s) is
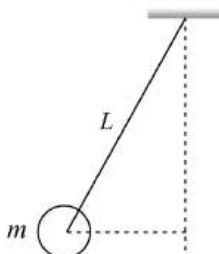

   a) 9 b) 18 c) 27 d) 36

21. Two identical balls A and B each of mass $0.1\ kg$ are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius $0.06\ m$. Each spring has a natural length of $0.06\pi\ m$ and force constant $0.1N/m$. Initially both the balls are

displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest. The frequency of oscillation of the ball B is

a) $\pi\ Hz$

b) $\dfrac{1}{\pi}Hz$

c) $2\pi\ Hz$

d) $\dfrac{1}{2\pi}Hz$

22. What is the maximum acceleration of the particle doing the SHM?

$y = 2\sin\left[\dfrac{\pi t}{2} + \emptyset\right]$ where 2 is in cm.

a) $\dfrac{\pi}{2}$ cms^{-2}

b) $\dfrac{\pi^2}{2}$ cms^{-2}

c) $\dfrac{\pi}{4}$ cms^{-2}

d) $\dfrac{\pi}{4}$ cms^{-2}

23. A particle moves according to the law, $= r\cos\dfrac{\pi t}{2.}$. The distance covered by it the time interval between t= 0 to t=3s is

a) r

b) 2r

c) 3r

d) 4r

24. How does the time period of pendulum vary with length

a) $\sqrt{L}$

b) $\sqrt{\dfrac{L}{2}}$

c) $\dfrac{1}{\sqrt{L}}$

d) $2L$

25. A force of 6.4 N stretches a vertical spring by 0.1 m. The mass that must be suspended from the spring so that it oscillates with a period of $\left(\dfrac{\pi}{4}\right)s.$ is

a) $\left(\dfrac{\pi}{4}\right)kg$

b) $1kg$

c) $\left(\dfrac{1}{\pi}\right)kg$

d) $10kg$

26. A metal rod of length L and mass m is pivoted at one end. A thin disk of mass M and radius $R(< L)$ is attached at its centre to the free end of the rod. Consider two ways the disc is attached **case A**- the disc is not free to rotate about its centre and **case B** – the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true?
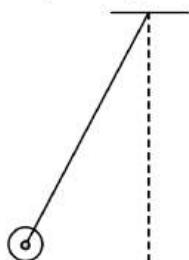

a) Restoring torque in case A=Restoring torque in case B

b) Restoring torque in case A<Restoring torque in case B

c) Angular frequency for case A<Angular frequency for case B

d) Angular frequency for case A<Angular frequency for case B

27. A man having a wrist watch and a pendulum clock rises on a TV tower. The wrist watch and pendulum clock by chance fall from the top of the tower. Then
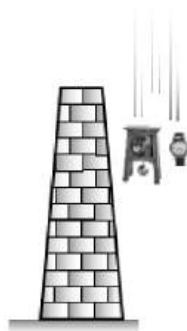
a) Both will keep correct time during the fall
b) Both will kept incorrect time during the fall
c) Wrist watch will keep correct time and clock will become fast
d) Clock will stop but wrist watch will function normally

28. For a particle executing SHM, the kinetic energy k is given by $k = k_0 \cos^2 \omega t$. The equation of its displacement can be

a) $\left(\dfrac{k_0}{m\omega^2}\right)^{1/2} \sin \omega t$
b) $\left(\dfrac{2k_0}{m\omega^2}\right)^{1/2} \sin \omega t$
c) $\left(\dfrac{2\omega^2}{m\,k_0}\right)^{1/2} \sin \omega t$
d) $\left(\dfrac{2k_0}{m\omega}\right)^{1/2} \sin \omega t$

29. As shown in figure, a simple harmonic motion oscillator having identical four springs has time period

a) $T = 2\pi\sqrt{\dfrac{m}{4k}}$
b) $T = 2\pi\sqrt{\dfrac{m}{2k}}$
c) $T = 2\pi\sqrt{\dfrac{m}{k}}$
d) $T = 2\pi\sqrt{\dfrac{2m}{k}}$

30. A particle of mass 200 g executes SHM. The restoring force is provided by a spring of force constant 80 N/m. The time period of oscillation is
a) 0.31 s
b) 0.15 s
c) 0.05 s
d) 0.02 s

31. The variation of potential energy of harmonic oscillator is as shown in figure. The spring constant is


a) $1 \times 10^2 N/m$
b) $150\ N/m$
c) $0.667 \times 10^2 N/m$
d) $3 \times 10^2 N/m$

32. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
a) First increase and then decrease to the origin value
b) First decrease and then increase to the origin value
c) Remain unchanged
d) Increase towards a saturation value

33. Length of a simple pendulum is l and its maximum angular displacement is θ, then its maximum K.E. is
a) $mgl \sin \theta$
b) $mgl(1 + \sin \theta)$
c) $mgl(1 + \cos \theta)$
d) $mgl(1 - \cos \theta)$

34. A simple pendulum has time period T. The bob is given negative charge and surface below it is given positive charge. The new time period will be
    a) Less than T b) Greater than T c) Equal to T d) Infinite

35. The displacement of a particle executing SHM is given by $y=0.25 \sin 200t$ cm. the maximum speed of the particle is
    a) 200 cms^{-1} b) 100 cms^{-1} c) 50 cms^{-1} d) 5.25 cms^{-1}

36. Graph between velocity and displacement of a particle, executing S.H.M. is
    a) A straight line   b) A parabola   c) A hyperbola   d) An ellipse

37. Displacement-time equation of a particle executing SHM is, $x = 4 \sin \omega t + 3 \sin(\omega t + \pi/3)$. Here x is in centimeter and t in second. The amplitude of oscillation of the particle is approximately
    a) 5 cm   b) 6 cm   c) 7 cm   d) 9 cm

38. A plate oscillates with time period $'T'$. Suddenly, another plate put on the first time, then time period
    a) Will decrease   b) Will increase   c) Will be same   d) None of these

39. A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance x. Now the combined mass will oscillate on the spring with period
    a) $T = 2\pi \sqrt{\dfrac{mg}{X(M + m)}}$ b) $T = 2\pi \sqrt{\dfrac{(M + m)X}{mg}}$

    c) $T = \pi/2 \sqrt{\dfrac{mg}{X(M + m)}}$ d) $T = 2\pi \sqrt{\dfrac{(M + m)}{mg}}$

40. An ideal spring with spring-constant K is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is
    a) $4 Mg/K$ b) $2 Mg/K$ c) Mg/K d) $Mg/2K$

41. Due to some force F_1 a body oscillates with period $4/5$ s and due to other force F_2 oscillates with period $3/5$ s. If both forces act simultaneously, the new period will be
    a) $0.72\ s$ b) $0.64\ s$ c) $0.48\ s$ d) $0.36\ s$

42. The time period of a mass suspended from a spring is 5 s. The spring is cut into four equal parts and the same mass is now suspended from one of its parts. The period is now
    a) 5 s   b) 2.5 s   c) 1.25 s   d) $\dfrac{1}{16}\ s$

43. A block of mass M is suspended from a light spring of force constant k. another mass m moving upwards with velocity v hits the mass M and gets embedded in it. What will be the amplitude of the combined mass?
    a) $\dfrac{mv}{\sqrt{(M - m)k}}$ b) $\dfrac{Mv}{(M - m)k}$ c) $\dfrac{mv}{\sqrt{(M + m)k}}$ d) $\dfrac{Mv}{\sqrt{(M + m)k}}$

44. A small block is connected to one end of a massless spring of un-stretched length $4.9m$. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by $0.2m$ and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} rad/s$. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P is at angle of $45°$ as shown in the figure. Point P is at a horizontal distance of $10\ m$ from O. If the pebble hits the block at $t = 1s$, the value of v is (take $g = 10m/s^2$)
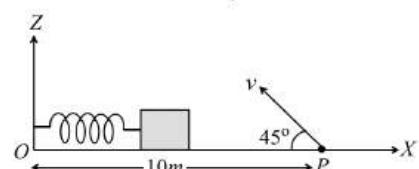
    a) $\sqrt{50}m/s$ b) $\sqrt{51}m/s$ c) $\sqrt{52}m/s$ d) $\sqrt{53}m/s$

45. The equation of motion of a particle is $\frac{d^2y}{dt^2} + Ky = 0$, where K is positive constant. The time period of the motion is given by

a) $\frac{2\pi}{K}$
b) $2\pi K$
c) $\frac{2\pi}{\sqrt{K}}$
d) $2\pi\sqrt{K}$

46. A particle executing a simple harmonic motion has a period of 6 s. The time taken by the particle to move from the mean position to half the amplitude, starting from the mean position is

a) $\frac{1}{4}$ s
b) $\frac{3}{4}$ s
c) $\frac{1}{2}$ s
d) $\frac{3}{2}$ s

47. A particle of mass m oscillates with simple harmonic motion between points x_1 and x_2, the equilibrium position being O. Its potential energy is plotted. It will be as given below in the graph
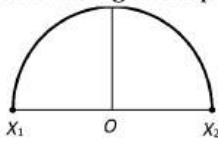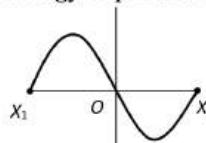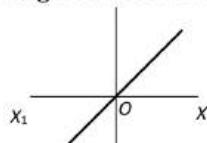
a)
b)
c)
d)
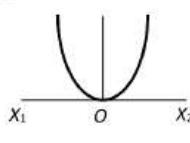
48. A particle is moving with constant angular velocity along the circumference of a circle. Which is the following statements is true

a) The particle so moving executes SHM
b) The projection of the particle of any one of the diameters executes SHM
c) The projection of the particle of any one of the diameters executes SHM
d) None of the above

49. A particle oscillating under a force $\vec{F} = -k\vec{x} - b\vec{v}$ is a (k and b are constant)

a) Simple harmonic oscillator
b) No linear oscillator
c) Damped oscillator
d) Forced oscillator

50. Infinite springs with force constants $k, 2k, 4k$ and $8k$ ….. respectively are connected in series. The effective force constant of the spring will be

a) $2k$
b) k
c) $k/2$
d) 2048

51. The period of oscillation of a simple pendulum of length l suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α is given by

a) $2\pi\sqrt{\frac{1}{g\cos\alpha}}$
b) $2\pi\sqrt{\frac{1}{g\sin\alpha}}$
c) $2\pi\sqrt{\frac{l}{g}}$
d) $2\pi\sqrt{\frac{1}{g\tan\alpha}}$

52. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is
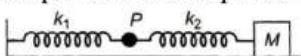
a) $\frac{k_1 A}{k_2}$
b) $\frac{k_2 A}{k_1}$
c) $\frac{k_1 A}{k_1 + k_2}$
d) $\frac{k_2 A}{k_1 + k_2}$

53. The time period of a simple pendulum of length L as measured in an elevator descending with acceleration $\frac{g}{3}$ is

a) $2\pi\sqrt{\frac{3L}{g}}$
b) $\pi\sqrt{\left(\frac{3L}{g}\right)}$
c) $2\pi\sqrt{\left(\frac{3L}{2g}\right)}$
d) $2\pi\sqrt{\left(\frac{2L}{3g}\right)}$

54. A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest its time period is T. With what acceleration should the lift be accelerated upwards in order to reduce its period to $T/2$? (g is acceleration due to gravity)

a) 2 g
b) 3 g
c) 4 g
d) g

55. Two simple pendulums of lengths 1.44 m and 1 m start swinging together. After how many vibrations will they again start swinging together?

a) 5 oscillations of smaller pendulum   b) 6 oscillations of smaller pendulum
c) 4 oscillations of bigger pendulum   d) 6 oscillations of bigger pendulum

56. The frequency of oscillation of the springs shown in the figure will be
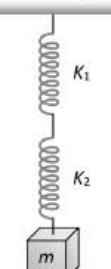
a) $\dfrac{1}{2\pi}\sqrt{\dfrac{K}{m}}$ b) $\dfrac{1}{2\pi}\sqrt{\dfrac{(K_1 + K_2)m}{K_1 K_2}}$ c) $2\pi\sqrt{\dfrac{K}{m}}$ d) $\dfrac{1}{2\pi}\sqrt{\dfrac{K_1 K_2}{m(K_1 + K_2)}}$

57. When a body of mass 1.0 kg is suspended from a certain light spring hanging vertically, its length increases by 5 cm. by suspending 2.0 kg block to the spring and if the block is pulled through 10 cm and released, the maximum velocity in it (in ms^{-1}) is (acceleration due to gravity=10 ms^{-2})

a) 0.5   b) 1   c) 2   d) 4

58. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constant k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is
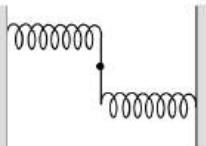
a) $\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{M}}$ b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{M}}$ c) $\dfrac{1}{2\pi}\sqrt{\dfrac{6k}{M}}$ d) $\dfrac{1}{2\pi}\sqrt{\dfrac{24k}{M}}$

59. The resultant of two rectangular single harmonic motion of the same frequency and unequal amplitudes but differing in phase by $\pi/2$ is

a) Simple harmonic   b) Circular   c) Elliptical   d) Parabolic

60. A block of mass m, attached to a spring of spring constant k, oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance x from the mean position, then

a) $x = \sqrt{m/k}$ b) $x = \dfrac{1}{v}\sqrt{m/k}$ c) $x = v\sqrt{m/k}$ d) $x = \sqrt{mv/k}$

61. Two identical blocks A and B, each of mass m resting on smooth floor, are connected by a light spring of natural length L and the spring constant k, with the spring at its natural length. A third identical block at C (mass m) moving with a speed (v) along the line joining A and B collides with A. The maximum compression in the spring is proportional to

a) $v\sqrt{\dfrac{m}{2k}}$ b) $m\sqrt{\dfrac{v}{2k}}$ c) $\sqrt{\dfrac{mv}{k}}$ d) $\dfrac{mv}{2k}$

62. The length of a spring is l and its force constant is k. When a weight W is suspended from it, its length increases by x. If the spring is cut into two equal parts and put in parallel and the same weight W is suspended from them, then the extension will be

a) $2x$ b) x c) $\dfrac{x}{2}$ d) $\dfrac{x}{4}$

63. A simple pendulum of length l has been set up inside a railway wagon sliding down a frictionless inclined plane having an angle of inclination $\theta = 30^0$ with the horizontal. What will be its period of oscillation as recorded by an observer inside the wagon?

   a) $2\pi\sqrt{\dfrac{2l}{\sqrt{3}\,g}}$ b) $2\pi\sqrt{2l/g}$ c) $2\pi\sqrt{l/g}$ d) $2\pi\sqrt{\dfrac{\sqrt{3}l}{2g}}$

64. Which of the following equations does not represent a simple harmonic motion
   a) $y = a\sin\omega t$ b) $y = a\cos\omega t$
   c) $y = a\sin\omega t + b\cos\omega t$ d) $y = a\tan\omega t$

65. A body is executing simple harmonic motion with an angular frequency $2\ rad/s$. The velocity of the body at $20\ mm$ displacement, when the amplitude of motion is $60\ mm$, is
   a) $40\ mm/s$ b) $60\ mm/s$ c) $113\ mm/s$ d) $120\ mm/s$

66. A piece of wood has dimensions a, b and c. Its relative density is d. It is floating in water such that the side c is vertical. It is now pushed down gently and released. The time period is

   a) $T = 2\pi\sqrt{\left(\dfrac{abc}{g}\right)}$ b) $T = 2\pi\sqrt{\left(\dfrac{b\,a}{dg}\right)}$ c) $T = 2\pi\sqrt{\left(\dfrac{g}{dc}\right)}$ d) $T = 2\pi\sqrt{\left(\dfrac{a\,c}{g}\right)}$

67. The metallic bob of a simple pendulum has the relative density ρ. The time period of this pendulum is T. If the metallic bob is immersed in water, then the new time period is given by

   a) $T\dfrac{\rho-1}{\rho}$ b) $T\dfrac{\rho}{\rho-1}$ c) $T\sqrt{\dfrac{\rho-1}{\rho}}$ d) $T\sqrt{\dfrac{\rho}{\rho-1}}$

68. A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to go directly from its mean position to half the amplitude
   a) $T/2$ b) $T/4$ c) $T/8$ d) $T/12$

69. A simple harmonic oscillator has a period T and energy E. the amplitude of the oscillator is doubled. Choose the correct answer.
   a) Period and energy get doubled   b) Period gets doubled while energy remains the same
   c) Energy gets double while period remains the same   d) Period remains the same and energy becomes four times

70. On a planet a freely falling body takes $2\ s$ when it is dropped from a height of $8\ m$, the time period of simple pendulum of length $1\ m$ on that planet is
   a) $3.14\ s$ b) $16.28\ s$ c) $1.57\ s$ d) None of these

71. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = k\,t^2, (k = 1\ \text{ms}^{-2})$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\dfrac{T_1^2}{T_2^2}$ is $(g = 10\ \text{ms}^{-2})$
   a) $6/5$ b) $5/6$ c) 1 d) $4/5$

72. A particle of mass m is located in a one dimensional potential field where potential energy is given by $(x) = A(1 - \cos px)$, where A and p are constants. The period of small oscillations of the particle is

   a) $2\pi\sqrt{\dfrac{m}{A\,p}}$ b) $2\pi\sqrt{\dfrac{m}{A\,p^2}}$ c) $2\pi\sqrt{\dfrac{m}{A}}$ d) $\dfrac{1}{2\pi}\sqrt{\dfrac{AR}{m}}$

73. An object is attached to the bottom of a light vertical spring and set vibrating. The maximum speed of the object is $15\ cm/s$ and the period is $628\ milli\text{-}seconds$. The amplitude of the motion in centimeters is
   a) 3.0 b) 2.0 c) 1.5 d) 1.0

74. The length of the second's pendulum is decreased by 0.3 cm when it is shifted to Chennai from London. If the acceleration due to gravity at London is 981 cms^{-2}, the acceleration due to gravity at Chennai is (assume $\pi^2 = 10$)

    a) 981 cms^{-2} b) 978 cms^{-2} c) 984 cms^{-2} d) 975 cms^{-2}

75. The velocity of a particle performing simple harmonic motion, when it passes through its mean position is

    a) Infinity   b) Zero   c) Minimum   d) Maximum

76. A girl swings on cradle in a sitting position. If she stands what happens to the time period of girl and cradle?

    a) Time period decreases   b) Time period increases

    c) Remains constant   d) First increases and then remains constant

77. For a simple pendulum, the graph between T^2 and L is

    a) A straight line passing through the origin   b) Parabola

    c) Circle   d) Ellipse

78. The motion which is not simple harmonic is

    a) Vertical oscillations of a spring   b) Motion of simple pendulum

    c) Motion of a planet around the sun   d) Oscillation of liquid column in a U-tube

79. In a simple harmonic oscillator, at the mean position

    a) Kinetic energy is minimum, potential energy is maximum

    b) Both kinetic and potential energies are maximum

    c) Kinetic energy is maximum, potential energy is minimum

    d) Both kinetic and potential energies are minimum

80. Which of the following figure represent(s) damped simple harmonic motions?
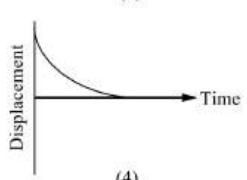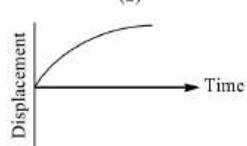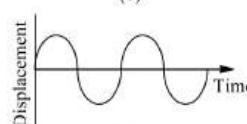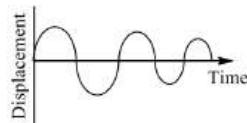
    a) Fig. 1 alone   b) Fig. 2 alone   c) Fig. 4 alone   d) Fig. 3 and 4

81. The bob of a simple pendulum is of mass 10 g. It is suspended with a thread of 1 m. If we hold the bob so as to stretch the string horizontally and release it, what will be the tension at the lowest position? (g= 10 ms^{-2})

    a) zero   b) 0.1 N   c) 0.3 N   d) 1.0 N

82. A body of mass 20 g connected to spring of constant k executes simple harmonic motion with a frequency of $\left(\frac{5}{\pi}\right)$ Hz. The value of spring constant is

    a) 4 Nm^{-1} b) 3 Nm^{-1} c) 2 Nm^{-1} d) 5 Nm^{-1}

83. Two SHMs are respectively represented by $y_1 = a \sin(\omega t - kx)$ and

$y_2 = b\cos(\omega t - kx)$ The phase difference between the two is

a) $\pi/6$　　　　　　　b) $\pi/4$　　　　　　　c) $\pi/2$　　　　　　　d) π

84. A mass M is attached to a horizontal spring of force constant k fixed on one side to a rigid support as shown in figure. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position. Another mass m is gently placed on it. What will be the new amplitude of oscillations?
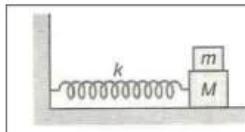

a) $A\sqrt{\left(\dfrac{M}{M-m}\right)}$　　　b) $A\sqrt{\left(\dfrac{M-m}{M}\right)}$　　　c) $A\sqrt{\left(\dfrac{M}{M+m}\right)}$　　　d) $A\sqrt{\left(\dfrac{M+m}{M}\right)}$

85. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period become $5T/3$. Then the ratio of $\dfrac{m}{M}$ is

a) $3/5$　　　　　　　b) $25/9$　　　　　　　c) $16/9$　　　　　　　d) $5/3$

86. Five identical springs are used in the following three configurations. The time periods of vertical oscillations in configurations (i), (ii) and (iii) are in the ratio
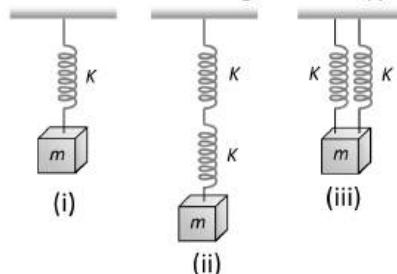
a) $1 : \sqrt{2} : \dfrac{1}{\sqrt{2}}$　　b) $2 : \sqrt{2} : \dfrac{1}{\sqrt{2}}$　　c) $\dfrac{1}{\sqrt{2}} : 2 : 1$　　d) $2 : \dfrac{1}{\sqrt{2}} : 1$

87. Two simple harmonic motions are represented by $y_1 = 4\sin(4\pi t + \frac{\pi}{2})$ and $y_2 = 3\cos(4\pi t)$. The resultant amplitude is

a) 7　　　　　　　b) 1　　　　　　　c) 5　　　　　　　d) $2 + \sqrt{3}$

88. Two simple pendulums first of bob mass M_1 and length L_1 second of bob mass M_2 and length L_2. $M_1 = M_2$ and $L_1 = 2L_2$. If the vibrational energy of both is same. Then which is correct

a) Amplitude of B greater than A　　　　b) Amplitude of B smaller than A
c) Amplitude will be same　　　　　　　　d) None of these

89. Four massless springs whose force constants are $2k, 2k, k$ and $2k$ respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of oscillation of the system is
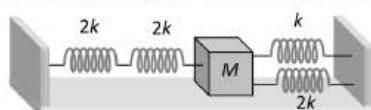
a) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{4M}}$　　b) $\dfrac{1}{2\pi}\sqrt{\dfrac{4k}{M}}$　　c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{7M}}$　　d) $\dfrac{1}{2\pi}\sqrt{\dfrac{7k}{M}}$

90. A particle moving along the x-axis executes simple harmonic motion, then the force acting on it is given by

a) $-A\,Kx$　　　　b) $A\cos(Kx)$　　　　c) $A\exp(-Kx)$　　　　d) $A\,Kx$

91. The period of a simple pendulum inside a stationary lift is T. The lift accelerates upwards with an acceleration of $g/3$. The time period of pendulum will be

a) $\sqrt{2}\, T$ b) $\dfrac{T}{\sqrt{2}}$ c) $\dfrac{\sqrt{3}}{2}T$ d) $\dfrac{T}{3}$

92. A particle is performing simple harmonic motion along x-axis with amplitude 4 cm and time period 1.2 s. The minimum time taken by the particle to move from $x = +2$ to $x = +4$ cm and back again is given by
   a) 0.4 s   b) 0.3 s   c) 0.2 s   d) 0.6 s

93. A body having natural frequency v' is executing forced oscillations under a driving force of frequency. The system will vibrate
   a) with frequency of driving force v b) with its natural frequency v'
   c) with mean frequency of the two $[(v + v')/2$ d) None of the above

94. The amplitude of a particle executing SHM is made three-fourth keeping its time period constant. Its total energy will be
   a) $\dfrac{E}{2}$ b) $\dfrac{3}{4}E$ c) $\dfrac{9}{16}E$ d) None of these

95. A wooden cube (density of wood d) of slide l floats in a liquid of density ρ with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, its performs simple harmonics motion of period T, then T is equal
   a) $2\pi\sqrt{\dfrac{l\rho}{(\rho - d)g}}$ b) $2\pi\sqrt{\dfrac{ld}{\rho g}}$ c) $2\pi\sqrt{\dfrac{l\rho}{dg}}$ d) $2\pi\sqrt{\dfrac{ld}{(\rho - d)g}}$

96. The restoring force of SHM is maximum when particle
   a) Displacement is maximum   b) Is half way between the mean and extreme position
   c) Crosses mean position   d) Is at rest

97. A particle is oscillating in SHM. What fraction of total energy is kinetic when the particle is at $A/2$ from the mean position? (A is the amplitude of oscillation)
   a) $\dfrac{3E}{2}$ b) $\dfrac{3}{4}E$ c) $\dfrac{E}{2}$ d) $3E$

98. The displacement equation of a simple harmonic oscillator is given by
   $$y = A\sin \omega t - B\cos \omega t$$
   The amplitude of the oscillator will be
   a) $A - B$ b) $A + B$ c) $\sqrt{A^2 + B^2}$ d) $A^2 + B^2$

99. What is the effect on the time period of a simple pendulum if the mass of the bob is doubled
   a) Halved   b) Doubled   c) Becomes eight times   d) No effect

100. The time period of a particle in simple harmonic motion is 8 seconds. At $t = 0$, it is at the mean position. The ratio of the distances travelled by it in the first and second seconds is
   a) 1/2   b) $1/\sqrt{2}$ c) $1/(\sqrt{2} - 1)$ d) $1/\sqrt{3}$

101. A system exhibiting S.H.M. must possess
   a) Inertia only   b) Elasticity as well as inertia
   c) Elasticity, inertia and an external force   d) Elasticity only

102. A simple pendulum of length l has a brass bob attached at its lower end. Its period is T. If a steel bob of same size, having density x times that of brass, replaces the brass bob and its length is changed so that period becomes 2 T. then new length is
   a) $2\,l$ b) $4\,l$ c) $4\,lx$ d) $4\,l/x$

103. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
   a) $T = t_1 + t_2$ b) $T^2 = t_1^2 + t_2^2$ c) $T^{-1} = t_1^{-1} + t_2^{-1}$ d) $T^{-2} = t_1^{-2} + t_2^{-2}$

104. A bottle weighing 220 g and of area of cross-section 50 cm² and height 4 cm oscillates on the surface of water in vertical position. Its frequency of oscillation is
   a) 1.5 Hz b) 2.5 Hz c) 3.5 Hz d) 4.5 Hz

105. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = \frac{4}{3}$ s is
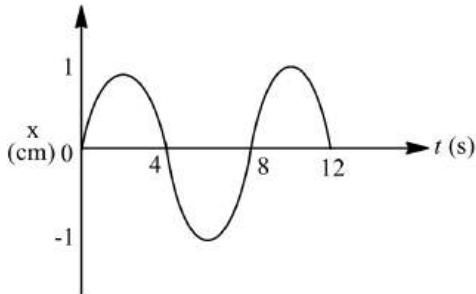
   a) $\frac{\sqrt{3}}{32}\pi^2 \text{cms}^{-2}$ b) $-\frac{\pi^2}{32}\text{cms}^{-2}$ c) $\frac{\pi^2}{32}\text{cms}^{-2}$ d) $-\frac{\sqrt{3}}{32}\pi^2\text{cms}^{-1}$

106. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is
   a) 0.5 s b) 0.75 s c) 0.125 s d) 0.25 s

107. A 15 g ball is shot from a spring gun whose spring has a force constant of 600 N/m. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression is ($g = 10\ m/s^2$)
   a) 6.0 m b) 10.0 m c) 12.0 m d) 8.0 m

108. The acceleration a of a particle undergoing S.H.M. is shown in the figure. Which of the labelled points corresponds to the particle being at $-x_{max}$
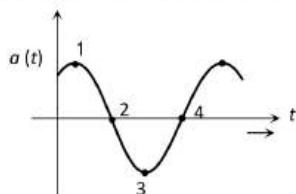
   a) 4 b) 3 c) 2 d) 1

109. The velocity-time diagram of a harmonic oscillator is shown in the adjoining figure. The frequency of oscillation is
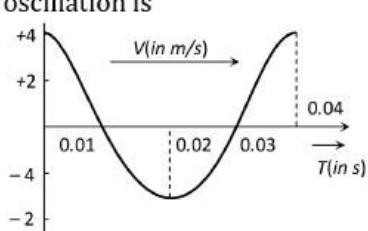
   a) 25 Hz b) 50 Hz c) 12.25 Hz d) 33.3 Hz

110. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi\sqrt{\frac{1}{g'}}$, where g' is equal to
   a) g b) $g - a$ c) $g + a$ d) $\sqrt{g^2 + a^2}$

111. A tunnel is made across the earth of radius R, passing through its centre. A ball is dropped from a height h in the tunnel. The motion will be periodic with time period.

a) $2\pi \sqrt{\dfrac{R}{g}} + 4\sqrt{\dfrac{h}{g}}$

b) $2\pi \sqrt{\dfrac{R}{g}} + 4\sqrt{\dfrac{2h}{g}}$

c) $2\pi \sqrt{\dfrac{R}{g}} + \sqrt{\dfrac{h}{g}}$

d) $2\pi \sqrt{\dfrac{R}{g}} + \sqrt{\dfrac{2h}{g}}$

112. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will
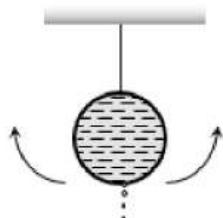
a) Remains unchanged
b) Increase
c) Decrease
d) Become erratic

113. If the differential equation given by

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = F_0 \sin pt$$

Describes the oscillatory motion of body in a dissipative medium under the influence of a periodic force, then the state of maximum amplitude of the oscillation is a measure of

a) Free vibration   b) Damped vibration   c) Forced vibration   d) Resonance

114. A simple pendulum is released from A shown.
If m and l represent the mass of the bob and
Length of the pendulum, the gain kinetic energy
at B is
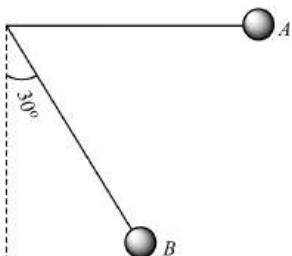
a) $\dfrac{mgl}{2}$

b) $\dfrac{mgl}{\sqrt{2}}$

c) $\dfrac{\sqrt{3}}{2}mgl$

d) $\dfrac{2}{\sqrt{3}}mgl$

115. A point mass m is suspended at the end of a massless wire of length L and cross-section area A. If Y is the Young's modulus for the wire, then the frequency of oscillations for the SHM along the vertical line is

a) $\dfrac{1}{2\pi}\sqrt{\dfrac{YA}{mL}}$

b) $2\pi\sqrt{\dfrac{mL}{YA}}$

c) $\dfrac{1}{\pi}\sqrt{\dfrac{YA}{mL}}$

d) $\pi\sqrt{\dfrac{mL}{YA}}$

116. The graph between the time period and the length of a simple pendulum is

a) Straight line   b) Curve   c) Ellipse   d) Parabola

117. A coin is placed on a horizontal platform, which undergoes horizontal SHM about a mean position O. the coin placed on platform dies not slip, coefficient of friction between the coin and the platform isμ. The amplitude of oscillation is gradually increased. The coin will begin to slip on the platform for the first time

a) at the mean position   b) at the extreme position of oscillations

c) for an amplitude of $\mu g/\omega^2$ d) for an amplitude of $g/\mu\,\omega^2$

118. The displacement of a particle performing simple harmonic motion is given by, $x = 8\sin\omega t + 6\cos\omega t$, where distance is in cm and time is in second. The amplitude of motion is
     a) 10 cm   b) 2 cm   c) 14 cm   d) 3.5 cm

119. The S.H.M. of a particle is given by the equation $y = 3\sin\omega t + 4\cos\omega t$. The amplitude is
     a) 7   b) 1   c) 5   d) 12

120. A particle executes SHM of amplitude 25 cm and time period 3 s. What is the minimum time required for the particle to move between two points 12.5 cm on either side of the mean position?
     a) 0.5 s   b) 1.0 s   c) 1.5 s   d) 2.0 s

121. A particle in SHM is described by the displacement function $x(t) = A\cos(\omega t + \phi)$, $\omega = 2\pi/T$. If the initial (t=0) position of the particle is 1 cm, its initial velocity is $\pi\ cm\ s^{-1}$ and its angular frequency is πs^1, then the amplitude of its motion is
     a) πcm b) 2 cm   c) $\sqrt{2}$ cm   d) 1 cm

122. Two springs are joined and attached to a mass of 16 kg. This system is then suspended vertically from a rigid support. The spring constant of the two springs are k_1 and k_2 respectively. The period of vertical oscillations of the system will be
     a) $\dfrac{1}{8\pi}\sqrt{k_1 + k_2}$ b) $8\pi\sqrt{\dfrac{k_1 + k_2}{k_1 k_2}}$ c) $\dfrac{\pi}{2}\sqrt{k_1 - k_2}$ d) $\dfrac{\pi}{2}\sqrt{\dfrac{k_1}{k_2}}$

123. A 0.10 kg block oscillates back and forth along a horizontal surface. Its displacement from the origin is given by: $x = (10cm)\cos[(10rad/s)t + \pi/2rad]$. What is the maximum acceleration experienced by the block
     a) $10\ m/s^2$ b) $10\ \pi\ m/s^2$ c) $\dfrac{10\pi}{2}m/s^2$ d) $\dfrac{10\pi}{3}m/s^2$

124. A pendulum has time period T. If it is taken on to another planet having acceleration due to gravity half and mass 9 times that of the earth then its time period on the other planet will be
     a) $\sqrt{T}$ b) T c) $T^{1/3}$ d) $\sqrt{2}\ T$

125. A particle executes SHM with a period of 8 s and amplitude 4 cm. Its maximum speed in cms^{-1}, is
     a) π b) $\dfrac{\pi}{2}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{\pi}{4}$

126. The time period of the variation of potential energy of a particle executing SHM with period T is
     a) $\dfrac{T}{4}$ b) T c) $2T$ d) $\dfrac{T}{2}$

127. A particle executes SHM in a line 4 cm long. Its velocity when passing through the centre of line is 12 cm/s. The period will be
     a) 2.047 s b) 1.047 s c) 3.047 s d) 0.047 s

128. A spring having a spring constant $'K'$ is loaded with a mass $'m'$. The spring is cut into two equal parts and one of these is loaded again with the same mass. The new spring constant is
     a) $K/2$ b) K c) $2K$ d) K^2

129. The KE and PE of a particle executing SHM of amplitude a will be equal when displacement is
     a) $\dfrac{a}{2}$ b) $a\sqrt{2}$ c) $2a$ d) $a/\sqrt{2}$

130. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π result in the displacement of the particle along
     a) circle   b) figure of eight   c) straight line   d) ellipse

131. A particle of mass m is hanging vertically by an ideal spring of force constant K. If the mas is made to oscillate vertically, its total energy is
     a) Maximum at extreme position   b) Maximum at mean position
     c) Minimum at mean position   d) Same at all position

132. A particle moves in $x - y$ plane according to rule $x = a \sin \omega t$ and $y = a \cos \omega t$. The particle follows
    a) An elliptical path
    b) A circular path
    c) A parabolic path
    d) A straight line path inclined equally to x and $y-$ axis

133. If the displacement of a particle executing SHM is given by $y = 0.30 \sin(220t + 0.64)$ in $metre$, then the frequency and maximum velocity of the particle is
    a) $35\ Hz, 66\ m/s$
    b) $45\ Hz, 66\ m/s$
    c) $58\ Hz, 113\ m/s$
    d) $35\ Hz, 132\ m/s$

134. Two SHMs are represented by the equations $y_1 = 0.1 \sin(100\pi t + \pi/3$ and $y_2 = 0.1 cos 100\pi t$. The phase difference of velocity of particle 2 with respect to the velocity of particle 1 is
    a) $-\pi/3$
    b) $\pi/6$
    c) $-\pi/6$
    d) $\pi/3$

135. A uniform cylinder of length L and mass M having cross sectional area A is suspended with its vertical length, from a fixed point by a massless spring, such that it is half submerged in a liquid of density d at equilibrium position. When released, it starts oscillating vertically with a small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is
    a) $\dfrac{1}{2\pi}\left(\dfrac{k - A\,d\,g}{M}\right)^{1/2}$
    b) $\dfrac{1}{2\pi}\left(\dfrac{k + A\,d\,g}{M}\right)^{1/2}$
    c) $\dfrac{1}{2\pi}\left(\dfrac{k - d\,g\,L}{M}\right)^{1/2}$
    d) $\dfrac{1}{2\pi}\left(\dfrac{k + A\,g\,L}{A\,d\,g}\right)^{1/2}$

136. In case of a simple pendulum, time period versus length is depicted by

    a)
    b)
    c)
    d)
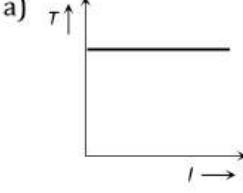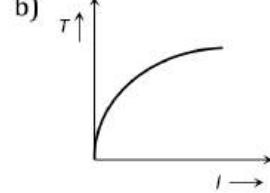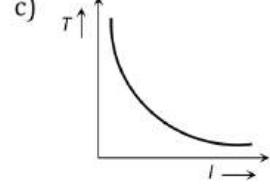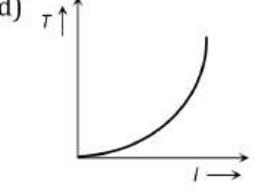
137. If a body oscillates at the angular frequency ω_d of the driving force, then the oscillations are called
    a) Free oscillations
    b) Coupled oscillations
    c) Forced oscillations
    d) Maintained oscillations

138. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as: $a = A \cos(\omega t + \delta)$, then
    a) $A = x_0, \delta = -\pi/4$
    b) $A = x_0 \omega^2, \delta = \pi/4$
    c) $A = x_0 \omega^2, \delta = -\pi/4$
    d) $A = x_0 \omega^2, \delta = 3\pi/4$

139. The amplitude of an oscillating simple pendulum is $10 cm$ and its period is $4\ s$. Its speed after $1\ s$ after it passes its equilibrium position, is
    a) Zero
    b) $0.57 m/s$
    c) $0.212 m/s$
    d) $0.32 m/s$

140. The acceleration of a particle performing SHM is $12\ \mathrm{cms^{-2}}$ at a distance of 3 cm from the mean position. Its time period is
    a) 2.0 s
    b) 3.14 s
    c) 0.5 s
    d) 1.0 s

141. Time period of a block suspended from the upper plate of a parallel plate capacitor by a spring of stiffness k is T. When block is uncharged. If a charge q is given to the block then, the new time period of oscillation will be
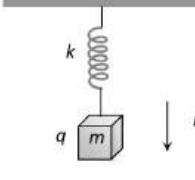
    a) T
    b) $> T$
    c) $< T$
    d) $\geq T$

142. Two points are located at a distance of $10\ m$ and $15\ m$ from the source of oscillation. The period of oscillation is $0.05\ sec$ and the velocity of the wave is $300\ m/sec$. What is the phase difference between the oscillations of two points

a) π b) $\dfrac{\pi}{6}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{2\pi}{3}$

143. A particle executes simple harmonic motion with an amplitude $4\ cm$. At the mean position the velocity of the particle is $10\ cm/s$. The distance of the particle from the mean position when its speed becomes $5\ cm/s$ is

a) $\sqrt{3}\ cm$ b) $\sqrt{5}\ cm$ c) $2(\sqrt{3})cm$ d) $2(\sqrt{5})cm$

144. A simple pendulum is hanging from a peg inserted in a vertical wall. Its bob is stretched in horizontal position from the wall and is left free to move. The bob hits on the wall the coefficient of restitution is $\dfrac{2}{\sqrt{5}}$. After how many collisions the amplitude of vibration will become less than $60°$

a) 6 b) 3 c) 5 d) 4

145. A particle of amplitude A is executing simple harmonic motion. When the potential energy of particle is half of its maximum potential energy, then displacement from its equilibrium position is

a) $\dfrac{A}{4}$ b) $\dfrac{A}{3}$ c) $\dfrac{A}{2}$ d) $\dfrac{A}{\sqrt{2}}$

146. A simple pendulum oscillates in air with time period T and amplitude A. As the time passes

a) T and A both decreases b) T increases and A is constant
c) T remains same and A decreses d) T decreases and A is constant

147. The period of particle in linear SHM is 8 s. At t=0, it is at the mean position. The ratio of the distances travelled by it in Its second and 2nd second is

a) $1.6 : 1$ b) $2.4 : 1$ c) $3.2 : 1$ d) $4.2 : 1$

148. A pendulum clock is placed on the moon, where object weighs only one-sixth as much as on earth, how many seconds the clock tick out in an actual time of 1 minute the clock keeps good time on earth?

a) 12.25 b) 24.5 c) 2.45 d) 0.245

149. Two bodies M and N of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude M to that of N is

a) k_1/k_2 b) $\sqrt{k_1/k_2}$ c) k_2/k_1 d) $\sqrt{k_2/k_1}$

150. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes slightly and released the mass executes a simple harmonic motion. The spring constant is 200 Nm^{-1}. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan? (Take g=10 ms^{-2})
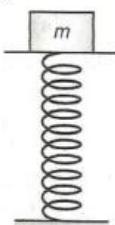

a) 8.0 cm b) 10.0 cm
c) Any value less than 12.0 cm d) 4.0 cm

151. Choose the correct statement :

a) Time period of a simple pendulum depends on amplitude
b) Time shown by a spring watch varies with acceleration due to gravity

c) In a simple pendulum time period varies linearly with the length of the pendulum

d) The graph between length of the pendulum and time period is a parabola

152. Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillations is
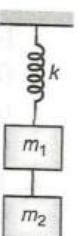
a) $\dfrac{m_1 g}{k}$ b) $\dfrac{m_2 g}{k}$ c) $\dfrac{(m_1 + m_2)g}{k}$ d) $\dfrac{(m_1 - m_2)g}{k}$

153. Two pendulums of length 1 m and 16 m start vibrating one behind the other from the same stand. At some instant, the two are in the mean position in the same phase. The time period of shorter pendulum is T. the minimum time after which the two threads of the pendulum will be one behind the other is?

a) $T/4$ b) $T/3$ c) $4\,T/3$ d) $4\,T$

154. In simple harmonic motion, the ratio of acceleration of the particle to its displacement at any time is a measure of

a) Spring constant   b) Angular frequency   c) (Angular frequency)^2 d) Restoring force

155. Two particles P and Q start from origin and execute Simple Harmonic Motion along X-axis with same amplitude but with period 3 $seconds$ and 6 $seconds$ respectively. The ratio of the velocities of the velocities of P and Q when they meet is

a) $1:2$ b) $2:1$ c) $2:3$ d) $3:2$

156. A light spiral spring supports a 200 g weight at its lower end. It oscillates up and down with a period of 1 s. How much weight (in gram) must be removed from the lower end to reduce the period to 0.5 s?

a) 53   b) 100   c) 150   d) 200

157. A pendulum of length 1 m is released from $\theta = 60°$. The rate of change of speed of the bob at $\theta = 30°$ is $(g = 10 \text{ ms}^{-2})$

a) 10 ms^{-2} b) 7.5 ms^{-2} c) 5 ms^{-2} d) $5\sqrt{3} \text{ ms}^{-2}$

158. A simple pendulum of length l and mass (bob) m is suspended vertically. The string makes an angle θ with the vertical. The restoring force acting on the pendulum is

a) $mg \tan \theta$ b) $-mg \sin \theta$ c) $mg \sin \theta$ d) $-mg \cos \theta$

159. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as function of displacement x. Which of the following statement is true?

a) KE is maximum when $x=0$ b) TE is zero when $x=0$

c) KE is maximum when x is maximum   d) PE is maximum when $x=0$

160. The height of a swing changes during its motion from 0.1 m to 2.5 m. The minimum velocity of a boy who swings in this swing is

a) $5.4\ m/s$ b) $4.95\ m/s$ c) $3.14\ m/s$ d) Zero

161. The displacement x (in $metres$) of a particle performing simple harmonic motion is related to time t (in $seconds$) as $x = 0.05 \cos\left(4\pi t + \dfrac{\pi}{4}\right)$. The frequency of the motion will be

a) $0.5\ Hz$ b) $1.0\ Hz$ c) $1.5\ Hz$ d) $2.0\ Hz$

162. A particle is executing SHM at mid-point of mean position and extremely. What is the potential energy in terms of total energy (E)?

a) $\dfrac{E}{4}$ b) $\dfrac{E}{16}$ c) $\dfrac{E}{2}$ d) $\dfrac{E}{8}$

163. A pendulum has time period T in air. When it is made to oscillate in water, it acquired a time period $T' = \sqrt{2}T$. The density of the pendulum bob is equal to (density of water $= 1$)

a) $\sqrt{2}$ b) 2 c) $2\sqrt{2}$ d) None of these

164. Time period of a simple pendulum of length l is T_1 and time period of a uniform rod of the same length l pivoted about one end and oscillating in a vertical plane is T_2. Amplitude of oscillations in both the cases is small. Then T_1/T_2 is

a) $\dfrac{1}{\sqrt{3}}$ b) 1 c) $\sqrt{\dfrac{4}{3}}$ d) $\sqrt{\dfrac{3}{2}}$

165. A particle of mass m is attached to three identical springs A, B and C each of force constant k a shown in figure. If the particle of mass m is pushed slightly against the spring A and released then the time period of oscillations is
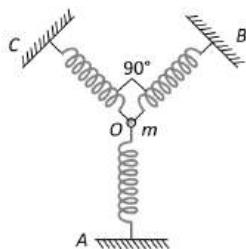
a) $2\pi\sqrt{\dfrac{2m}{k}}$ b) $2\pi\sqrt{\dfrac{m}{2k}}$ c) $2\pi\sqrt{\dfrac{m}{k}}$ d) $2\pi\sqrt{\dfrac{m}{3k}}$

166. For Simple Harmonic Oscillator, the potential energy is equal to kinetic energy

a) Once during each cycle b) twice during each cycle

c) when $x = a/2$ d) when $x = a$

167. The displacement of a particle in SHM various according to the relation $x = 4 (\cos\pi t + \sin\pi t)$. The amplitude of the particle is

a) -4 b) 4 c) $4\sqrt{2}$ d) 8

168. The maximum speed of a particle executing SHM is 1 ms^{-1} and maximum acceleration is 1.57 ms^{-2}. Its frequency is

a) $0.25\ s^1$ b) $2\ s^1$ c) $1.57\ s^1$ d) $2.57\ s^1$

169. A body of mass 5 g is executing SHM about a fixed point O. with an amplitude of 10 cm, its maximum velocity is 100 cms^{-1}. Its velocity will be 50 cms^{-1} at a distance (in cm)

a) 5 b) $5\sqrt{2}$ c) $5\sqrt{3}$ d) $10\sqrt{2}$

170. In the figure, S_1 and S_2 are identical springs. The oscillation frequency of the mass m is f. If one spring is removed, the frequency will become
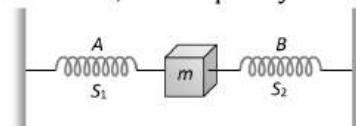
a) f b) $f \times 2$ c) $f \times \sqrt{2}$ d) $f/\sqrt{2}$

171. A particle of mass m is executing oscillations about the origin on the x-axis with amplitude A. Its potential energy $U(x) = ax^4$ where a is positive constant. The x-coordinate of mass where potential energy is one-third of the kinetic energy of particle is

a) $\dfrac{\pm A}{\sqrt{3}}$ b) $\dfrac{\pm A}{\sqrt{2}}$ c) $\dfrac{\pm A}{3}$ d) $\dfrac{\pm A}{2}$

172. The displacement of tow particles executing SHM are represented equations $y_1 = 2\sin(10t + \theta)$, $y_2 = 3\cos 10t$. The phase difference between the velocity of these particles is

    a) θ b) $-\theta$ c) $\theta + \pi/2$ d) $\theta - \pi/2$

173. Starting from $y = A\sin\omega t$ or $y = A\cos\omega t$

    a) acceleration lags the displacement by a phase $\pi/4$ b) acceleration lags the displacement by a phase $\pi/2$

    c) acceleration leads the displacement by a phase $\pi/2$ d) acceleration leads the displacement by a phase π

174. For any S.H.M. amplitude is $6\ cm$. If instantaneous potential energy is half the total energy then distance of particle from its mean position is

    a) $3\ cm$ b) $4.2\ cm$ c) $5.8\ cm$ d) $6\ cm$

175. The displacement y in cm is given in terms of time t sec by the equation

y = 3 sin 314t + cos314t

The amplitude of SHM is

    a) 7 cm   b) 3 cm   c) 4 cm   d) 5 cm

176. A particle is executing S.H.M. Then the graph of acceleration as a function of displacement is

    a) A straight line   b) A circle   c) A ellipse   d) A hyperbola

177. Two particles execute SHM of the same amplitude and frequency along the same straight line. If they pass one another when going in opposite directions, each time their displacement is half their amplitude, the phase difference between them is

    a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{6}$ d) $\dfrac{2\pi}{3}$

178. Two springs with spring constants $K_1 = 1500\ N/m$ and $K_2 = 3000 N/m$ are stretched by the same force. The ratio of potential energy stored in spring will be

    a) 2 :1   b) 1 :2   c) 4 :1   d) 1 :4

179. If the period of oscillation of mass m suspended from a spring is 2 s, then the period of mass $4m$ will be

    a) $1\ s$ b) $2\ s$ c) $3\ s$ d) $4\ s$

180. If the length of second's pendulum is increased by 2% How many seconds it will lose per day?

    a) 3927 s   b) 3427 s   c) 3737 s   d) 864 s

181. If a simple pendulum of length, L has maximum angular displacement α, then the maximum kinetic energy of bob of mass M is

    a) $\dfrac{1}{2}\dfrac{ML}{g}$ b) $\dfrac{Mg}{2L}$ c) $MgL(1 - \cos\alpha)$ d) $\dfrac{MgL\sin\alpha}{2}$

182. The angular velocities of three bodies in simple harmonic motion are $\omega_1, \omega_2, \omega_3$ with their respective amplitudes as $A_1, A_2, A_3,$. If all the bodies have same mass and velocity, then

    a) $A_1\omega_1 = A_2\omega_2 = A_3\omega_3$ b) $A_1\omega_1{}^2 = A_2\omega_2{}^2 = A_3\omega_3{}^2$

    c) $A_1{}^2\omega_1 = A_2{}^2\omega_2 = A_3{}^2\omega_3$ d) $A_1{}^2\omega_1{}^2 = A_2{}^2\omega_2{}^2 = A^2$

183. Time period of mass m suspended by a spring is T. If the spring is cut to one-half and made to oscillate by suspending double mass, the time period of the mass will be

    a) 8T   b) 4T   c) $\dfrac{T}{2}$ d) T

184. The springs shown are identical. When $A = 4kg$, the elongation of spring is $1\ cm$. If $B = 6kg$, the elongation produced by it is
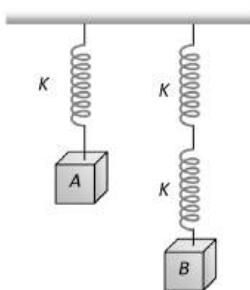
a) 4 cm   b) 3 cm   c) 2 cm   d) 1 cm

185. A mass m is suspended from the two coupled springs connected in series. The force constant for springs are K_1 and K_2. The time period of the suspended mass will be

a) $T = 2\pi \sqrt{\left(\dfrac{m}{K_1 + K_2}\right)}$

b) $T = 2\pi \sqrt{\left(\dfrac{2m}{K_1 + K_2}\right)}$

c) $T = 2\pi \sqrt{\left(\dfrac{m(K_1 + K_2)}{K_1 K_2}\right)}$

d) $T = 2\pi \sqrt{\left(\dfrac{mK_1 K_2}{K_1 + K_2}\right)}$

186. A horizontal platform with an object placed on it is executing SHM in the vertical direction. The amplitude of oscillation is $30.92 \times 10^{-3} m$. What must be the least period of these oscillations, so that the object is not detached from the platform?

a) 0.1256 s   b) 0.1356 s   c) 0.1456 s   d) 0.1556 s

187. A disc of radius R and mass M is pivoted at the rim and is set for small oscillations. If simple pendulum has to have the same period as that of the disc, the length of the simple pendulum should be

a) $\dfrac{5}{4}R$ b) $\dfrac{2}{3}R$ c) $\dfrac{3}{4}R$ d) $\dfrac{3}{2}R$

188. A body of mass 0.01 kg executes SHM about $x = 0$, under the influence of force shown in the figure The period of the SHM is
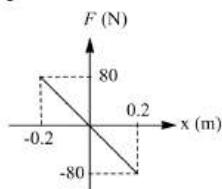
a) 1.05 s   b) 0.52 s   c) 0.25 s   d) 0.03 s

189. A particle is moving with constant angular velocity along the circumference of a circle. Which of the following statements is true
a) The particle so moving executes S.H.M.
b) The projection of the particle on any one of the diameters executes S.H.M.
c) The projection of the particle on any of the diameters executes S.H.M.
d) None of the above

190. Two identical pendulum are oscillating with amplitudes 4 cm and 8 cm. the ratio of their energies of oscillation will be

a) 1/3   b) 1/4   c) 1/9   d) 1/2

191. Consider the following statements :
The total energy of a particle executing simple harmonic motion depends on its
I.   Amplitude
II. Period
III. Displacement
Of these statements
a) I and II are correct   b) II and III are correct

c) I and III are correct

d) I , II and III are correct

192. The total energy of a particle, executing simple harmonic motion is

Where x is the displacement from the mean position.

a) $\propto x$

b) $\propto x^2$

c) Independent of x

d) $\propto x^{1/2}$

193. The bob of a pendulum of length l is pulled aside from its equilibrium position through an angle θ and then released. The bob will then pass through its equilibrium position with a speed v, where v equals

a) $\sqrt{2gl(1 - \cos\theta)}$

b) $\sqrt{2gl(1 + \sin\theta)}$

c) $\sqrt{2gl(1 - \sin\theta)}$

d) $\sqrt{2gl(1 + \cos\theta)}$

194. While driving around a curve of 200 m radius the driver noted that pendulum in the car hangs at an angle of 15^0 to the vertical. The speedometer of the car reads (in ms^{-1})

a) 20

b) 23

c) 230

d) 236

195. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ

a) $\sqrt{2}A, \frac{3\pi}{4}$

b) $A, \frac{4\pi}{3}$

c) $\sqrt{3}A, \frac{5\pi}{6}$

d) $A, \frac{\pi}{3}$

196. A mass M is suspended by two springs of force constants K_1 and K_2 respectively as shown in the diagram. The total elongation (stretch) of the two springs is
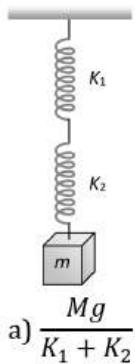
a) $\dfrac{Mg}{K_1 + K_2}$

b) $\dfrac{Mg(K_1 + K_2)}{K_1 K_2}$

c) $\dfrac{MgK_1 K_2}{K_1 + K_2}$

d) $\dfrac{K_1 + K_2}{K_1 K_2 Mg}$

197. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

a) $2\pi\left(\dfrac{m}{K}\right)$

b) $2\pi\left\{\dfrac{(YA + KL)m}{YAK}\right\}^{1/2}$

c) $2\pi\dfrac{mYA}{KL}$

d) $2\pi\dfrac{mL}{YA}$

198. What is constant in S.H.M.

a) Restoring force

b) Kinetic energy

c) Potential energy

d) Periodic time

199. A simple pendulum has a time period T in vacuum. Its time period when it is completely immersed in a liquid of density one-eight of the density of material of the bob is

a) $\sqrt{\dfrac{7}{8}}T$

b) $\sqrt{\dfrac{5}{8}}T$

c) $\sqrt{\dfrac{3}{8}}T$

d) $\sqrt{\dfrac{8}{7}}T$

200. Two springs of force constant k_1 and k_2 are connected as shown.
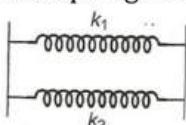
The effective spring constant k is

a) $k_1 + k_2$

b) $\dfrac{k_1}{k_2}$

c) $k_1 k_2$

d) $2k_1 k_2$

201. A particle of mass 1 kg is moving in SHM with an amplitude 0.02 m and a frequency of 60 Hz. The maximum force in newton acting on the particle is
   a) $188\,\pi^2$
   b) $144\pi^2$
   c) $288\pi^2$
   d) None of these

202. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 m, is $4.4\ \text{ms}^{-1}$. The period of oscillation is
   a) 0.01 s
   b) 10 s
   c) 0.1 s
   d) 100 s

203. A body executing simple harmonic motion has a maximum acceleration equal to $24\ \text{ms}^{-2}$ and maximum velocity of $16\ \text{ms}^{-1}$, the amplitude of the simple harmonic motion is
   a) $\frac{1024}{9}$ m
   b) $\frac{32}{3}$ m
   c) $\frac{64}{9}$ m
   d) $\frac{3}{32}$m

204. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is (where E is the total energy)
   a) $\frac{1}{8}E$
   b) $\frac{1}{4}E$
   c) $\frac{1}{2}E$
   d) $\frac{2}{3}E$

205. The minimum phase difference between two simple harmonic oscillations,

   $$y_1 = \frac{1}{2}\sin \omega t + \frac{\sqrt{3}}{2}\cos \omega t$$

   $$y_2 = \sin \omega t + \cos \omega t,\ \text{is}$$
   a) $\frac{7\pi}{12}$
   b) $\frac{\pi}{12}$
   c) $-\frac{\pi}{6}$
   d) $\frac{\pi}{6}$

206. The length of simple pendulum is increased by 1%. Its time period will
   a) increase by 2%
   b) increase by 1%
   c) increase by 0.5%
   d) decrease by 0.5%

207. In case of a forced vibration, the resonance wave becomes very sharp when the
   a) Restoring force is small
   b) Applied periodic force is small
   c) Quality factor is small
   d) Damping force is small

208. The pendulum bob has a speed of 3ms^{-1} at its lowest position. The pendulum is 0.5 m long. The speed of the bob, when the length makes an angle of 60° to the vertical will be ($g = 10\ \text{ms}^{-2}$)
   a) $\frac{1}{2}\ \text{ms}^{-1}$
   b) $\frac{1}{3}\ \text{ms}^{-1}$
   c) $3\ \text{ms}^{-1}$
   d) $2\ \text{ms}^{-1}$

209. If a simple pendulum is taken to a place where g decreases by 2% then the time period
   a) increases by 0.5%
   b) increases by 1%
   c) increases by 2.0%
   d) decreases by 0.5%

210. Time period of a spring mass system is T. If this spring is cut into two parts whose lengths are in the ratio 1:3 and the same mass is attached to the longer part, the new time period will be
   a) $\sqrt{\frac{3}{2}}\ T$
   b) $\frac{T}{\sqrt{3}}$
   c) $\frac{\sqrt{3}T}{2}$
   d) $\sqrt{3}T$

211. A weightless spring of length 60 cm and force constant 200 N/m is kept straight and unstretched on a smooth horizontal table and its ends are rigidly fixed. A mass of 0.25 kg is attached at the middle of the spring and is slightly displaced along the length. The time period of the oscillation of the mass is
   a) $\frac{\pi}{20}s$
   b) $\frac{\pi}{10}s$
   c) $\frac{\pi}{5}s$
   d) $\frac{\pi}{\sqrt{200}}s$

212. Identify correct statement among the following
   a) The greater the mass of a pendulum bob, the shorter is its frequency of oscillation
   b) A simple pendulum with a bob of mass M swings with an angular amplitude of 40°. When its angular amplitude is 20°, the tension in the string is less than $Mg\cos 20°$.
   c) As the length of a simple pendulum is increased, the maximum velocity of its bob during its oscillation will decrease
   d) The fractional change in the time period of a pendulum on changing the temperature is independent of the length of the pendulum

213. The average acceleration of a particle performing SHM over one complete oscillation is

a) $\dfrac{\omega^2 A}{2}$ b) $\dfrac{\omega^2 A}{\sqrt{2}}$ c) Zero d) $A\omega^2$

214. A particle is moving in a circle with uniform speed. Its motion is
a) Periodic and simple harmonic
b) Periodic but no simple harmonic
c) A periodic
d) None of the above

215. The differential equation of a particle executing SHM along y-axis is

a) $\dfrac{d^2y}{dt^2} + \omega^2 y = 0$ b) $\dfrac{d^2y}{dt^2} + \omega^2 y^2 = 0$ c) $\dfrac{d^2y}{dt^2} - \omega^2 y = 0$ d) $\dfrac{d^2y}{dt^2} + \omega y = 0$

216. A simple harmonic oscillator has an amplitude a and time period T. The time required by it to travel from $x = a$ to $x = a/2$ is
a) $T/6$ b) $T/4$ c) $T/3$ d) $T/2$

217. If the length of a pendulum is made 9 times and mass of the bob is made 4 times then the value of time period becomes
a) $3T$ b) $3/2T$ c) $4T$ d) $2T$

218. Two simple pendulums whose lengths are $100cm$ and $121cm$ are suspended side by side. Their bobs are pulled together and then released. After how many minimum oscillations of the longer pendulum, will the two be in phase again
a) 11 b) 10 c) 21 d) 20

219. Resonance is an example of
a) Tuning fork b) Forced vibration c) Free vibration d) Damped vibration

220. When a mass m is attached to a spring, it normally extends by $0.2\ m$. The mass m is given a slight addition extension and released, then its time period will be

a) $\dfrac{1}{7} sec$ b) $1\ sec$ c) $\dfrac{2\pi}{7}\pi$ d) $\dfrac{2}{3\pi} sec$

221. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α is given by

a) $2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$ b) $2\pi\sqrt{\dfrac{L}{g\sin\alpha}}$ c) $2\pi\sqrt{\dfrac{L}{g}}$ d) $2\pi\sqrt{\dfrac{L}{g\tan\alpha}}$

222. Out of the following functions representing motion of a particle which represents SHM

(1) $y = \sin\omega t - \cos\omega t$ (2) $y = \sin^3\omega t$

(3) $y = 5\cos\left(\dfrac{3\pi}{4} - 3\omega t\right)$ (4) $y = 1 + \omega t + \omega^2 t^2$

a) Only (1) and (2) b) Only (1)
c) Only (4) does not represent SHM d) Only (1) and (3)

223. A simple pendulum has a length l. The inertial and gravitational masses of the bob are m_i and m_g respectively. Then the time period T is given by

a) $T = 2\pi\sqrt{\dfrac{m_g l}{m_i g}}$ b) $T = 2\pi\sqrt{\dfrac{m_i l}{m_g g}}$

c) $T = 2\pi\sqrt{\dfrac{m_i \times m_g \times l}{g}}$ d) $T = 2\pi\sqrt{\dfrac{l}{m_i \times m_g \times g}}$

224. The total energy of a simple harmonic oscillator is proportional to
a) Square root of displacement
b) Velocity
c) Frequency
d) Square of the amplitude

225. The displacement of a particle from its mean position (in metre) is given by $y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$. The motion of particle is
a) Periodic but not S.H.M.
b) Non-periodic

c) Simple harmonic motion with period 0.1 s

d) Simple harmonic motion with period 0.2 s

226. What will be the force constant of the spring system shown in figure?
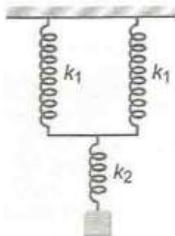
a) $\frac{k_1}{2} + k_2$

b) $\left[\frac{1}{2k_1} + \frac{1}{k_2}\right]^{-1}$

c) $\frac{1}{2k_1} + \frac{1}{k_2}$

d) $\left[\frac{2}{k_1} + \frac{1}{k_2}\right]^{-1}$

227. A particle is executing SHM of period $24x$ and of amplitude 41 cm with O as equilibrium position. The minimum time in seconds taken by the particle to go from P to Q. where OP $= -9$ cm and OQ $=40$ cm is

a) 5
b) 6
c) 7
d) 9

228. The velocity of particle in simple harmonic motion at displacement y from mean position is

a) $\omega\sqrt{a^2 + y^2}$
b) $\omega\sqrt{a^2 - y^2}$
c) ωy
d) $\omega^2\sqrt{a^2 - y^2}$

229. The ratio of frequencies of two pendulum are 2:3, then their lengths are in ratio

a) $\sqrt{2/3}$
b) $\sqrt{3/2}$
c) 4/9
d) 9/4

230. On a smooth inclined plane, a body of mass M is attached between two springs. The other ends of the springs are fixed to firm support. If each spring has force constant k, the period of oscillation of the body (assuming the springs as massless) is
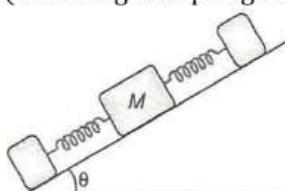
a) $2\pi[M/2k]^{1/2}$

b) $2\pi[2M/k]^{1/2}$

c) $2\pi[Mg\sin\theta/2k]^{1/2}$

d) $2\pi[2Mg/k]^{1/2}$

231. A body is vibrating in simple harmonic motion. If its acceleration is 12 cm s^{-2} at a displacement 3 cm, then time period is

a) 6.28 s
b) 3.14 s
c) 1.57 s
d) 2.57 s

232. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion

a) When v is maximum, a is maximum

b) Value of a is zero, whatever may be the value of v

c) When v is zero, a is zero

d) When v is maximum, a is zero

233. A body is moving in a room with a velocity of 20 m/s perpendicular to the two walls separated by 5 $meters$. There is no friction and the collisions with the walls are elastic. The motion of the body is

a) Not periodic

b) Periodic but not simple harmonic

c) Periodic and simple harmonic

d) Periodic with variable time period

234. The periodic time of a particle doing simple harmonic motion is 4 s. The taken by it to go from its mean position to half the maximum displacement (amplitude)

a) 2s
b) 1s
c) $\frac{2}{3}$s
d) $\frac{1}{3}$s

235. A uniform spring of force constant k is cut into two pieces, the lengths of which are in the ratio 1: 2. The ratio of the force constants of the shorter and longer piece is

a) 1 : 2
b) 2 : 1
c) 1 : 3
d) 2 : 3

236. A particle is executing simple harmonic motion with frequency f. The frequency at which its kinetic energy change into potential energy is

a) $f/2$ b) f c) $2f$ d) $4f$

237. A mass M, attached to a spring, Oscillates with a period of 2 s. If the mass is increased by 4 kg, the time period increases by 1 s. Assuming that Hooke's law is obeyed, the initial mass M was

    a) 3.2 kg   b) 1 kg   c) 2 kg   d) 8 kg

238. The kinetic energy and the potential energy of a particle executing S.H.M. are equal. The ratio of its displacement and amplitude will be

    a) $\dfrac{1}{\sqrt{2}}$ b) $\dfrac{\sqrt{3}}{2}$ c) $\dfrac{1}{2}$ d) $\sqrt{2}$

239. Which one of the following equations of motion represents simple harmonic motion

    Where k, k_0, k_1 and a are all positive

    a) Acceleration $= -k_0 x + k_1 x^2$ b) Acceleration $= -k(x + a)$

    c) Acceleration $= k(x + a)$ d) Acceleration $= kx$

240. Acceleration A and time period T of a body in S.H.M. is given by a curve shown below. Then corresponding graph, between identic energy (K.E) and time t is correctly represented by
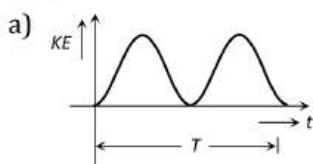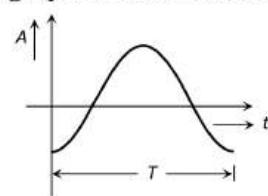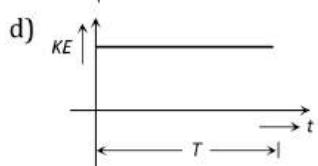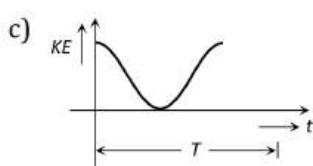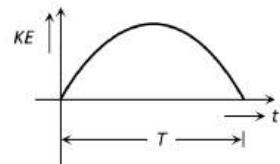
241. U is the PE of an oscillating particle and F is the force acting on it at a given instant. Which of the following is true?

    a) $\dfrac{U}{F} + x = 0$ b) $\dfrac{2U}{F} + x = 0$ c) $\dfrac{F}{U} + x = 0$ d) $\dfrac{F}{2U} + x = 0$

242. Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, and each time their displacement is half of their amplitude. The phase difference between them is

    a) 30°   b) 60°   c) 90°   d) 120°

243. A particle executes linear simple harmonic motion with an amplitude of 2 cm. When the particle is at 1 cm from the mean position the magnitude of its velocity is equal to that of its acceleration. Then its time period in second is

    a) $\dfrac{1}{2\pi\sqrt{3}}$ b) $2\pi\sqrt{3}$ c) $\dfrac{2\pi}{\sqrt{3}}$ d) $\dfrac{\sqrt{3}}{2\pi}$

244. To show that a simple pendulum executes simple harmonic motion, it is necessary to assume that

    a) Length of the pendulum is small   b) Mass of the pendulum is small

    c) Amplitude of oscillation is small   d) Acceleration due to gravity is small

245. A lift is ascending with an acceleration equal to g/3. Its time period of oscillation is T. What will be the time period of a simple pendulum suspended from its ceiling in stationary lift?

a) $2T$ b) $3T$ c) $(\sqrt{3/4})T$ d) $2T/\sqrt{3}$

246. If the displacement equation of a particle be represented by $y = A\sin PT + B\cos PT$, the particle executes
    a) A uniform circular motion
    b) A uniform elliptical motion
    c) A S.H.M.
    d) A rectilinear motion

247. A simple pendulum is set into vibrations. The bob of the pendulum comes to rest after some time due to
    a) Air friction
    b) Moment of inertia
    c) Weight of the bob
    d) Combination of all the above

248. A mass m attached to a spring oscillates every 2 s. If the mass is increased by 2 kg, then time-period increases by 1 s. The initial mass is
    a) 1.6 kg b) 3.9 kg c) 9.6 kg d) 12.6 kg

249. If a simple pendulum oscillates with an amplitude of 50 nm and time period of 2 s, then its maximum velocity is
    a) 0.10 ms^{-1} b) 0.15 ms^{-1} c) 0.8 ms^{-1} d) 0.26 ms^{-1}

250. The displacement of a particle executing SHM is given by $y = 5\sin\left(4t + \frac{\pi}{3}\right)$
    If T is the time period and mass of the particle is $2g$, the kinetic energy of the particle when $t = \frac{T}{4}$ is given by
    a) 0.4 J b) 0.5 J c) 3 J d) 0.3 J

251. Two identical springs are connected in series and parallel as shown in the figure. If f_s and f_p are frequencies of arrangements, what is $\frac{f_s}{f_p}$?
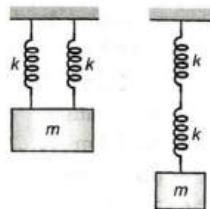
    a) 1:2   b) 2:1   c) 1:3   d) 3:1

252. The scale of a spring balance reading from 0 to 10 kg is 0.25 m long. A body suspended from the balance oscillates vertically with a period of $\pi/10$ second. The mass suspended is (neglect the mass of the spring)
    a) 10 kg b) 0.98 kg c) 5 kg d) 20 kg

253. A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system if a mass $4m$ is suspended from the same spring
    a) $n/4$ b) $4n$ c) $n/2$ d) $2n$

254. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly show in figure.

    a)   b)   c)   d)
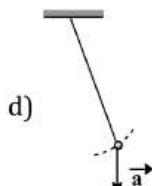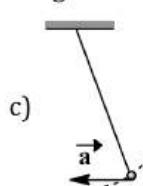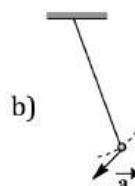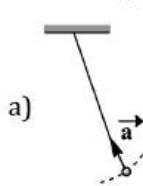
255. A particle starts SHM from the mean position. Its amplitude is a and total energy E. At one instant its kinetic energy is $3\frac{E}{4}$. Its displacement at that instant is
    a) $\frac{a}{\sqrt{2}}$ b) $\frac{a}{2}$ c) $\frac{a}{\sqrt{\left(\frac{3}{2}\right)}}$ d) $\frac{a}{\sqrt{3}}$

256. A man measures the period of a simple pendulum inside a stationary lift ad finds it to be T second. If the lift accelerates upwards with an acceleration g/4, then the period of pendulum will be

a) $2T\sqrt{5}$ b) T c) $\dfrac{2T}{\sqrt{5}}$ d) $\dfrac{T}{4}$

257. One end of a spring of force constant k is fixed to a vertical wall and the other to a block of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the black. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall is
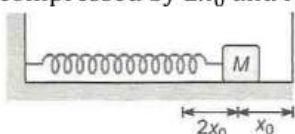
a) $\dfrac{1}{6}\pi\sqrt{\dfrac{k}{m}}$ b) $\sqrt{\dfrac{k}{m}}$ c) $\dfrac{2\pi}{3}\sqrt{\dfrac{m}{k}}$ d) $\dfrac{\pi}{4}\sqrt{\dfrac{k}{m}}$

258. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

a) $a^2T^2 + 4\pi^2 v^2$ b) $\dfrac{aT}{x}$ c) $aT + 2\pi v$ d) $\dfrac{aT}{v}$

259. In the figure, the vertical sections of the string are long. A is released from rest from the position shown. Then
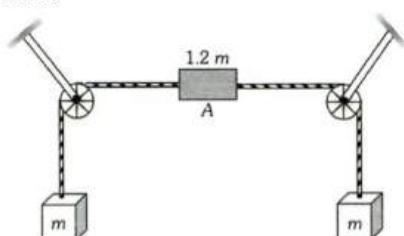
a) The system will remain in equilibrium
b) The central block will move down continuously
c) The central block will undergo simple harmonic motion
d) The central block will undergo periodic motion but not simple harmonic motion

260. A horizontal platform vibrates with simple harmonic motion in the horizontal direction with a period 2 s. A body of mass 0.5 kg is placed on the platform. The coefficient of static friction between the body and platform is 0.3. What is the maximum frictional force on the body when the platform is oscillating with amplitude 0.2 m? Assume $\pi^2 = 10 = g$.

a) 0.5 N b) 1 N c) 1.5 N d) 2 N

261. The P.E. of a particle executing SHM at a distance x from its equilibrium position is

a) $\dfrac{1}{2}m\omega^2 x^2$ b) $\dfrac{1}{2}m\omega^2 a^2$ c) $\dfrac{1}{2}m\omega^2(a^2 - x^2)$ d) Zero

262. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x.
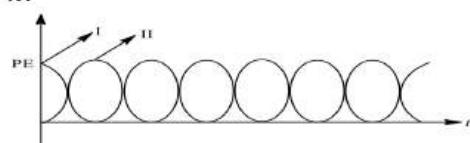

a) I, III b) II, IV c) II, III d) I, IV

263. For a particle in SHM, if the amplitude of the displacement is a and the amplitude of velocity is v' the amplitude of acceleration is
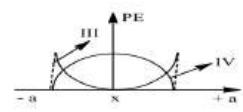
a) va b) $\dfrac{v^2}{a}$ c) $\dfrac{v^2}{2a}$ d) $\dfrac{v}{a}$

264. Two pendulums have time period T and 5T/4. They start SHM at the same time from the mean position. What will be the phase difference between then after the bigger pendulum completed one oscillation?

a) 45^0 b) 90^0 c) 60^0 d) 30^0

265. In a seconds pendulum, mass of the bob is 30 g. If it is replaced by 90 g mass, then its time period will be

a) 1 s   b) 2 s   c) 4 s   d) 3 s

266. The time period of a simple pendulum is 2 s. If its length is increased 4 times, then its period becomes

a) $16\ s$ b) $12\ s$ c) $8\ s$ d) $4\ s$

267. The periodic time of a body executing simple harmonic motion is 3 s. After how much interval from time $t = 0$, its displacement will be half of its amplitude

a) $\dfrac{1}{8}s$ b) $\dfrac{1}{6}s$ c) $\dfrac{1}{4}s$ d) $\dfrac{1}{3}s$

268. For a body of mass m attached to the spring, the spring factor is given by (ω, the angular frequency)

a) m/ω^2 b) $m\ \omega^2$ c) $m^2\omega$ d) $m^2\omega^2$

269. A body of mass $1kg$ is executing simple harmonic motion. Its displacement $y(cm)$ at t seconds is given by $y = 6\sin(100t + \pi/4)$. Its maximum kinetic energy is

a) $6\ J$ b) $18\ J$ c) $24\ J$ d) $36\ J$

270. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0s$ to $t = \tau s$, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with $'b'$ as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds

a) $0.693/b$ b) b c) $1/b$ d) $2/b$

271. What is time period of pendulum hanged in satellite?

(T is time period on earth)

a) Zero   b) T c) Infinite   d) $T/\sqrt{6}$

272. A mass m performs oscillations of period T when hanged by spring of force constant K. If spring is cut in two parts and arranged in parallel and same mass is oscillated by them, then the new time period will be
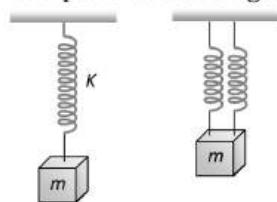
a) $2T$ b) T c) $\dfrac{T}{\sqrt{2}}$ d) $\dfrac{T}{2}$

273. A particle moves so that its acceleration a is given by $a = -bx$, where x is displacement from equilibrium position and b is a non-negative real constant. The time period of oscillation of the particle is

a) $2\pi\sqrt{b}$ b) $\dfrac{2\pi}{b}$ c) $\dfrac{2\pi}{\sqrt{b}}$ d) $2\sqrt{\dfrac{\pi}{b}}$

274. A simple pendulum hanging from the ceiling of a stationary lift has time period t_1. When the lift moves downward with constant velocity, the time period is t_2, then

a) t_2 is infinity   b) $t_2 > t_1$ c) $t_2 < t_1$ d) $t_2 = t_1$

275. A body of mass $500\ g$ is attached to a horizontal spring of spring constant $8\pi^2$ N m^{-1}. If the body is pulled to a distance of 10 cm from its mean position, then its frequency of oscillation is

a) $2\ Hz$ b) $4\ Hz$ c) $8\ Hz$ d) $0.5\ Hz$

276. The kinetic energy of a particle executing S.H.M. is $16\,J$ when it is at its mean position. If the mass of the particle is $0.32\,kg$, then what is the maximum velocity of the particle
   a) $5\,m/s$ b) $15\,m/s$ c) $10\,m/s$ d) $20\,m/s$

277. In SHM restoring force is $F = -k\,x$, where k is force constant, x is displacement and A is amplitude of motion, then total energy depends upon
   a) k, A and M b) k, x, M c) k, A d) k, x

278. To make the frequency double of a spring oscillator, we have to
   a) Reduce the mass to one fourth
   b) Quardruple the mass
   c) Double of mass
   d) Half of the mass

279. A particle of mass 10 g is executing simple harmonic motion with an amplitude of 0.5 m and periodic time of $(\pi/5)$ s. The maximum value of the force acting on the particle is
   a) 25 N   b) 5 N   c) 2.5 N   d) 0.5 N

280. A block whose mass is 650 g is fastened to a spring whose spring constantly is $65\ \mathrm{Nm^{-1}}$. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$. On a frictionless surface and released from rest at $t = 0$. The maximum velocity of the vibrating block is
   a) $1.1\ \mathrm{ms^{-1}}$ b) $0.65\ \mathrm{ms^{-1}}$ c) $1.30\ \mathrm{ms^{-1}}$ d) $2.6\ \mathrm{ms^{-1}}$

281.
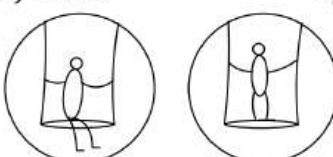
A child swings sitting and standing inside swing as shown in figure, then period of oscillations have the relation
   a) $(T)_{Sitting} = (T)_{Standing}$
   b) $(T)_{Sitting} > (T)_{Standing}$
   c) $(T)_{Sitting} < (T)_{Standing}$
   d) $2(T)_{Sitting} = (T)_{Standing}$

282. A particle is subjected simultaneously to two SHM's one along the x-axis and the other along the y-axis. The two vibrations are in phase and have unequal amplitudes. The particle will execute
   a) Straight line motion   b) Circular motion   c) Elliptic motion   d) Parabolic motion

283. A block is placed on a frictionless horizontal table. The mass of the block is m and springs are attached on either side with force constants K_1 and K_2. If the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be
   a) $\left(\dfrac{K_1 + K_2}{m}\right)^{1/2}$ b) $\left[\dfrac{K_1 K_2}{m(K_1 + K_2)}\right]^{1/2}$ c) $\left[\dfrac{K_1 K_2}{(K_1 - K_2)m}\right]^{1/2}$ d) $\left[\dfrac{K_1^2 + K_2^2}{(K_1 + K_2)m}\right]^{1/2}$

284. Two linear SHMs of equal amplitude A and angular frequencies ω and 2ω are impressed on a particle along the axes x and y recpectively. If the initial phase difference between them is $\pi/2$, the resultant path followed by the particle is
   a) $y^2 = x^2(1 - x^2/A^2)$
   b) $y^2 = 2x^2(1 - x^2 A^2)$
   c) $y^2 = 4x^2(1 - x^2/A^2)$
   d) $y^2 = 8x^2(1 - x^2/A^2)$

285. If a watch with a wound spring is taken on to the moon, it
   a) Runs faster   b) Runs slower   c) Does not work   d) Shown no change

286. The displacement x(in metre) of a particle in simple harmonic motion is related to time t(in second) as
$$x = 0.01 \cos\left(\pi t + \frac{\pi}{4}\right)$$
The frequency of the motion will be
   a) 0.5 Hz   b) 1.0 Hz   c) $\frac{\pi}{2}$ Hz   d) π Hz

287. A simple pendulum is attached to the roof of a lift. If time period of oscillation, when the lift is stationary is T. Then frequency of oscillation, when the lift falls freely, will be
   a) Zero   b) T c) $1/T$ d) None of these

288. A highly rigid cubical block A of small mass M and side L is fixed rigidly on the cubical block of same dimensions and low modulus of rigidity η such that the lower face of A completely covers the upper face of B. the lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. after the force is withdrawn, block A executes small oscillations, the time period of which is given by

a) $2\pi\sqrt{ML\eta}$
b) $2\pi\sqrt{M\eta/L}$
c) $2\pi\sqrt{ML/\eta}$
d) $2\pi\sqrt{M/\eta L}$

289. A heavy brass sphere is hung from a weightless inelastic spring and as a simple pendulum its time period of oscillation is T. When the sphere is immersed in a non-viscous liquid of density 1/10 that of brass, it will act as a simple pendulum of period

a) T
b) $\dfrac{10}{9}T$
c) $\sqrt{\left(\dfrac{9}{10}\right)}T$
d) $\sqrt{\left(\dfrac{10}{9}\right)}T$

290. If a simple harmonic is represented by $\dfrac{d^2x}{dt^2} + \alpha x = 0$, its time period is

a) $\dfrac{2\pi}{\alpha}$
b) $\dfrac{2\pi}{\sqrt{\alpha}}$
c) $2\pi\alpha$
d) $2\pi\sqrt{\alpha}$

291. A body of mass 4 kg hangs from a spring and oscillates with a period 0.5 s on the removel of the body, the spring is shortented by

a) 6.3 cm
b) 0.63 cm
c) 6.25 cm
d) 6.3 cm

292. Two particles A and B execute simple harmonic motion of period T and $5T/4$. They start from mean position. The phase difference between them when the particle A complete an oscillation will be

a) $\pi/2$
b) Zero
c) $2\pi/5$
d) $\pi/4$

293. A body of mass $8\ kg$ is suspended through two light springs X and Y connected in series as shown in figure. The readings in X and Y respectively are
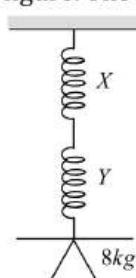
a) $8\ kg$, zero
b) zero, $8\ kg$
c) $8\ kg, 8\ kg$
d) $2\ kg, 6\ kg$

294. A simple pendulum is suspended from the ceiling of a stationary elevator and its period of oscillation is T. The elevator is then set into motion and the new time period is found to be longer. Then the elevator is

a) Accelerated upward
b) Accelerated downward
c) Moving downward with nonuniform speed
d) Moving downward with uniform speed

295. A spring (spring constant $=k$) is cut into 4 equal parts and two parts are connected in parallel. What is the effective spring constant?

a) $4\ k$
b) $16\ k$
c) $8\ k$
d) $6\ k$

296. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4 k, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\dfrac{y}{x}$ is
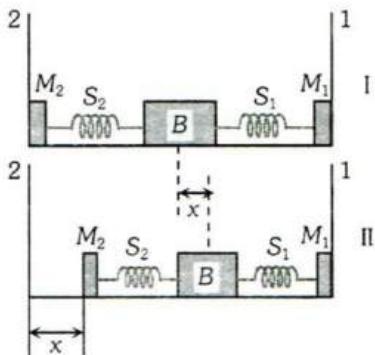
a) 4   b) 2   c) $\dfrac{1}{2}$ d) $\dfrac{1}{4}$

297. In S.H.M. maximum acceleration is at
   a) Amplitude
   b) Equilibrium
   c) Acceleration is constant
   d) None of these

298. A particle executing simple harmonic motion along y-axis has the its motion described by the equation $y = A\sin(\omega t) + B$. The amplitude of the simple harmonic motion is
   a) A b) B c) $A + B$ d) $\sqrt{A + B}$

299. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium is
   a) $\dfrac{T}{4}$ b) $\dfrac{T}{8}$ c) $\dfrac{T}{12}$ d) $\dfrac{T}{2}$

300. Maximum speed of a particle in SHM is v_{max}. Then average speed of a particle in SHM is equal to
   a) $\dfrac{v_{\text{max}}}{2}$ b) $\dfrac{\pi v_{\text{max}}}{2}$ c) $\dfrac{v_{\text{max}}}{2\pi}$ d) $\dfrac{2v_{\text{max}}}{\pi}$

301. In a simple harmonic oscillator has got a displacement of $0.02\ m$ and acceleration equal to $2.0 ms^{-2}$ at any time, the angular frequency of the oscillator is equal to
   a) $10 rad\ s^{-1}$ b) $0.1 rad\ s^{-1}$ c) $100 rad\ s^{-1}$ d) $1 rad\ s^{-1}$

302. A pendulum is made to hang from the ceiling of an elevator. It has period of T s (for small angles). The elevator is made to accelerate upwards with $10 m/s^2$. The period of the pendulum now will be (assume $g = 10 m/s^2$)
   a) $T\sqrt{2}$ b) Infinite   c) $T/\sqrt{2}$ d) Zero

303. A particle starts S.H.M. from the mean position. Its amplitude is A and time period is T. At the time when its speed is half of the maximum speed, its displacement y is
   a) $\dfrac{A}{2}$ b) $\dfrac{A}{\sqrt{2}}$ c) $\dfrac{A\sqrt{3}}{2}$ d) $\dfrac{2A}{\sqrt{3}}$

304. A particle is undergoing a one dimensional simple harmonic oscillation of amplitude X_M about the origin on X-axis with time period T and is at $-X_M$ at. What is the position of the particle after a time interval $t = 3.15\ T$?
   a) Between $-X_M$ and O
   b) Between O and $+X_M$
   c) At the origin
   d) At $+X_M$

305. A particle is having kinetic energy $1/3$ of the maximum value at a distance of 4 cm from the mean position, Find the amplitude of motion.
   a) $2\sqrt{6}\ cm$ b) $2/\sqrt{6}\ cm$ c) $\sqrt{2}\ cm$ d) $6\sqrt{2}\ cm$

306. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is
   a) -4   b) 4   c) $4\sqrt{2}$ d) 8

307. A particle is oscillating according to the equation $X = 7\cos 0.5\pi t$, where t is in second. The point moves from the position of equilibrium to maximum displacement in time

a) 4.0 s   b) 2.0 s   c) 1.0 s   d) 0.5 s

308. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000t)$. This expression may be considered to be a result of the superposition of........ independent harmonic motions
    a) Two   b) Three   c) Four   d) Five

309. A simple pendulum of length L and mass of bob M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement $|\theta| < \phi$. the tension in the string and the velocity of the bob are T and v respectively. Which of the following relation holds good under the above conditions?
    a) $T\cos\theta = Mg$ b) $T - Mg\cos\theta = Mv^2/L$
    c) $T + Mg\cos\theta = Mv^2/L$ d) $T = Mg\cos\theta$

310. A uniform rod of length 2.0 m is suspended through an end is set into oscillation with small amplitude under gravity. The time period of oscillation is approximately
    a) 1.60 s   b) 1.80 s   c) 2.0 s   d) 2.40 s

311. The displacement of a particle of mass 3 g executing simple harmonic motion is given by $Y = 3\sin(0.2t)$ in SI units. The KE of the particle at a point which is at a distance equal to 1/3 of its amplitude from its mean position is
    a) 12×10^{-3} J   b) 25×10^{-3} J   c) 0.48×10^{-3} J   d) 0.24×10^{-3} J

312. A uniform spring of force constant k is cut into two pieces whose lengths are in the ratio of 1:2. What is the force constant of second piece in terms of k?
    a) $\dfrac{k}{2}$ b) $\dfrac{2k}{2}$ c) $\dfrac{3k}{2}$ d) $\dfrac{4k}{2}$

313. An object suspended from a spring exhibits oscillations of period T. Now the spring is cut in two halves and the same object is suspended with two halves as shown in figure. The new time period of oscillation will become
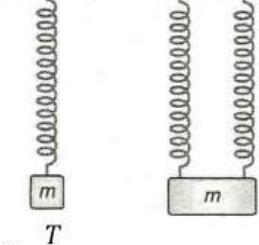
    a) $\dfrac{T}{2\sqrt{2}}$ b) $\dfrac{T}{2}$ c) $\dfrac{T}{\sqrt{2}}$ d) 2T

314. The displacement of two identical particle executing SHM are represented by equations $x_1 = 8\sin(10t + \pi/6)$ and $x_2 = 5\sin\omega t$.
    For what value of ω energy of both the particle is same?
    a) 4 units   b) 8 units   c) 16 units   d) 20 units

315. A particle performing SHM has time period $\dfrac{2\pi}{\sqrt{3}}$ and path length 4 cm. The displacement from mean position at which acceleration is equal to velocity is
    a) Zero   b) 0.5 cm   c) 1 cm   d) 1.5 cm

316. A simple harmonic motion is represented by $F(t) = 10\sin(20\,t + 0.5)$. The amplitude of the S.H.M. is
    a) $a = 30$ b) $a = 20$ c) $a = 10$ d) $a = 5$

317. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $x_0(x_0 > A)$. If the maximum separation between them is $(x_0 + A)$, the phase difference between their motions is
    a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{6}$ d) $\dfrac{\pi}{2}$

318. The time period of a simple pendulum, when it is made to oscillate on the surface of moon
    a) Increases   b) Decreases

c) Remains unchanged

d) Becomes infinite

319. A and B are fixed points and the mass M is tied by strings at A and B. If the mass M is displaced slightly out of this plane and released, it will execute oscillations with period (given $AM = BM = L$, $AB = 2d$)
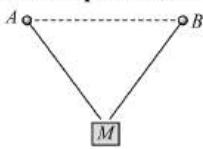
a) $2\sqrt{\dfrac{L}{g}}\pi$

b) $2\pi\sqrt{\dfrac{(L^2 - d^2)^{1/2}}{g}}$

c) $2\pi\sqrt{\dfrac{(L^2 + d^2)^{1/2}}{g}}$

d) $2\pi\sqrt{\dfrac{(2d^2)^{3/2}}{g}}$

320. The angular amplitude of a simple pendulum is θ_0. The maximum tension in its string will be

a) $mg(1 - \theta_0)$ b) $mg(1 + \theta_0)$ c) $mg(1 - \theta_0^2)$ d) $mg(1 + \theta_0^2)$

321. A weightless spring which has a force constant k oscillates with frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass 2 m is suspended from one part of spring. The frequency of oscillation will now become

a) N   b) 2n   c) $\dfrac{n}{\sqrt{2}}$ d) $n\,(2)^{1/2}$

322. Two pendulums of length 212 cm and 100 cm start vibrating. At same instant the two are in the mean position in the same phase. After how many vibrations of the shorter pendulum, the two will be in phase in the mean position?

a) 10   b) 11   c) 20   d) 21

323. The function $\sin^2(\omega t)$ represents

a) A periodic, but not simple harmonic, motion with a period $2\pi/\omega$

b) A periodic, but not simple harmonic, motion with a period π/ω

c) a simple harmonic motion with a period $2\pi/\omega$

d) a simple harmonic motion with a period π/ω

324. The periodic time of a simple pendulum of length $1\,m$ and amplitude $2\,cm$ is 5 seconds. If the amplitude is made $4\,cm$. Its periodic time in seconds will be

a) 2.5   b) 5   c) 10   d) $5\sqrt{2}$

325. The total energy of a particle executing S.H.M. is proportional to

a) Displacement from equilibrium position

b) Frequency of oscillation

c) Velocity in equilibrium position

d) Square of amplitude of motion

326. A hollow sphere is filled with water through the small hole in it. It is then hung by a long thread and made to oscillate. As the water slowly flow out of the hole at the bottom, the period of oscillation will

a) Continuously decrease

b) Continuously increase

c) First decrease then increase

d) First increase then decrease

327. A simple pendulum is set up in a trolley which moves to the right with an acceleration a on a horizontal plane. Then the thread of the pendulum in the mean position makes an angle θ with the vertical

a) $\tan^{-1}\dfrac{a}{g}$ in the forward direction

b) $\tan^{-1}\dfrac{a}{g}$ in the backward direction

c) $\tan^{-1}\dfrac{g}{a}$ in the backward direction

d) $\tan^{-1}\dfrac{g}{a}$ in the forward direction

328. Consider the mechanical vibrating systems shown in figure A, B, C and D. The vibrations are simple harmonic in
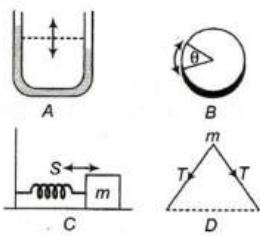
a) A, C b) A, B, C c) B, D d) A, B, C, D

329. A particle in SHM is described by the displacement function $x(t) = A \cos(\omega t + \theta)$. If the initial (t = 0) position of the particle is 1 cm and its initial velocity is πcms^{-1}, what is its amplitude? The angular frequency of the particle is $\pi\ s^{-1}$.

a) 1 cm   b) $\sqrt{2}\ cm$ c) 2 cm   d) 2.5 cm

330. The period of a simple pendulum is doubled, when

a) Its length is doubled

b) The mass of the bob is doubled

c) Its length is made four times

d) The mass of the bob and the length of the pendulum are doubled

331. A particle is executing simple harmonic motion with a period of $T\ seconds$ and amplitude a $metre$. The shortest time it takes to reach a point $\frac{a}{\sqrt{2}}m$ from its mean position in $seconds$ is

a) T b) $T/4$ c) $T/8$ d) $T/16$

332. A simple pendulum with a bob of mass $'m'$ oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is $'g'$, then the velocity of the bob as it passes through B is
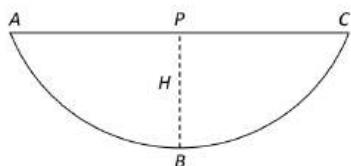
a) mgH b) $\sqrt{2gH}$ c) $2gH$ d) Zero

333. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k[x]^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

a) Proportional to $\frac{1}{\sqrt{a}}$ b) Independent to a c) Proportional to $\sqrt{a}$ d) Proportional to $a^{3/2}$

334. A particle is performing simple harmonic motion with amplitude A and angular velocity ω. The ratio of maximum velocity to maximum acceleration is

a) ω b) $1/\omega$ c) ω^2 d) $A\omega$

335. If $<E>$ and $<U>$ denote the average kinetic and the average potential energies respectively of mass describing a simple harmonic motion, over one period, then the correct relation is

a) $<E> = <U>$ b) $<E> = 2<U>$ c) $<E> = -2<U>$ d) $<E> = -<U>$

336. A pendulum of length $2m$ lift at P. When it reaches Q, it losses 10% of its total energy due to air resistance. The velocity at Q is
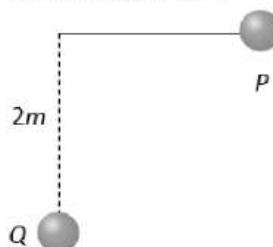
a) $6\ m/s$
b) $1\ m/s$
c) $2\ m/s$
d) $8\ m/s$

337. A particle executing simple harmonic motion with amplitude of 0.1 m. At a certain instant when its displacement is 0.02 m, its acceleration is 0.5 m/s^2. The maximum velocity of the particle is (in m/s)
   a) 0.01   b) 0.05   c) 0.5   d) 0.25

338. When a mass M is attached to the spring of force constant k, then the spring stretches by l. If the mass oscillates with amplitude l, what will be maximum potential energy stored in the spring
   a) $\dfrac{kl}{2}$ b) $2kl$ c) $\dfrac{1}{2}Mgl$ d) Mgl

339. A brass cube of side a and density σ is floating in mercury of density ρ. If the cube is displaced a bit vertically, it executes S.H.M. Its time period will be
   a) $2\pi\sqrt{\dfrac{\sigma a}{\rho g}}$ b) $2\pi\sqrt{\dfrac{\rho a}{\sigma g}}$ c) $2\pi\sqrt{\dfrac{\rho g}{\sigma a}}$ d) $2\pi\sqrt{\dfrac{\sigma g}{\rho a}}$

340. A simple spring has length l and force constant k. It is cut into two spring of length l_1 and l_2 such that $l_1 = nl_2$ (n = an integer). the force constant of the spring of length l_2 is
   a) $k(1+n)$ b) $(k/n)(1+n)$ c) k d) $k/(n+1)$

341. Which one of the following is a simple harmonic motion
   a) Wave moving through a string fixed at both ends
   b) Earth spinning about its own axis
   c) Ball bouncing between two rigid vertical walls
   d) Particle moving in a circle with uniform speed

342. The period of oscillation of a mass m suspended from a spring is 2 s. If along with it another mass 2 kg is also suspended, the period of oscillation increases by 1 s. the mass m will be
   a) 2 kg   b) 1 kg   c) 1.6 kg   d) 2.6 kg

343. The total energy of a simple harmonic oscillator is proportional to
   a) Square root of displacement   b) Velocity
   c) Frequency   d) Square of the amplitude

344. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T. Then
   a) $T \propto \dfrac{1}{\rho}$ b) $T \propto \dfrac{1}{\sqrt{m}}$ c) $T \propto \sqrt{\rho}$ d) $T \propto \dfrac{1}{\sqrt{A}}$

345. The amplitude of a damped oscillator becomes $\left(\dfrac{1}{3}\right)$ rd in 2 s. If its amplitude after 6 s is $\dfrac{1}{n}$ times the original amplitude, the value of n is
   a) 3^2 b) $3\sqrt{2}$ c) $3\sqrt{3}$ d) 3^3

346. A particle of mass m is released from rest and follows a parabolic path as shown. Assuming that the displacement of the mass from the origin is small, which graph correctly depicts the position of the particle as a function of time

   a)   b)   c)   d)
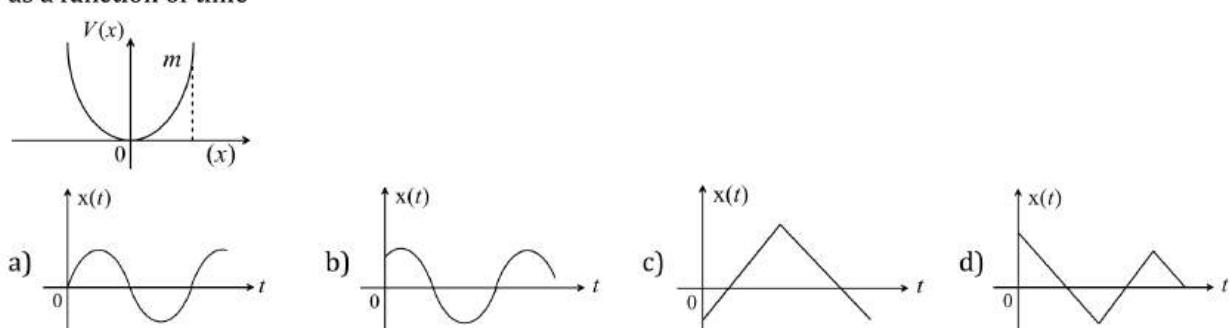
347. The amplitude of a particle executing SHM is 4 cm. At the mean position the speed of the particle is $16\,cms^{-1}$. The distance of the particle from the mean position at which the speed of the particle become $8\sqrt{3}\,cms^{-1}$, will be
   a) $2\sqrt{3}$ cm   b) $\sqrt{3}$ cm   c) 1 cm   d) 2 cm

348. The maximum velocity of a simple harmonic motion represented by $y = 3\sin(100t + \frac{\pi}{6})$ m is given by

a) $300\ ms^{-1}$
b) $\frac{3\pi}{6}\ ms^{-1}$
c) $100\ ms^{-1}$
d) $\frac{\pi}{6}\ ms^{-1}$

349. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then

a) $T_1 < T_2$
b) $T_1 > T_2$
c) $T_1 = T_2$
d) $T_1 = 2T_2$

350. The variation of the acceleration a of the particle executing S.H.M. with displacement y is as shown in the figure

a)

b)

c)

d)
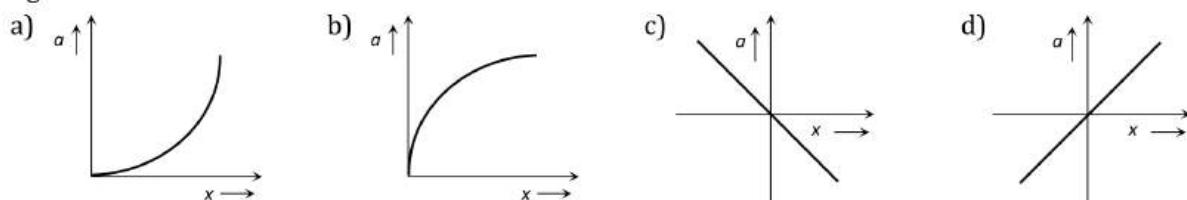
351. Values of the acceleration A of a particle moving in simple harmonic motion as a function of its displacement x are given in the table below

$A(mm\ s^{-2})$	16	8	0	-8	-16
$x\ (mm)$	-4	-2	0	2	4

The period of the motion is

a) $\frac{1}{\pi}s$
b) $\frac{2}{\pi}s$
c) $\frac{\pi}{2}s$
d) $\pi\ s$

352. The bob of a simple pendulum of length L is released at time t = 0 from a position of small angular displacement. Its linear displacement at time t is given by

a) $X = a\sin 2\pi\sqrt{\frac{L}{g}} \times t$

b) $X = a\cos 2\pi\sqrt{\frac{g}{L}} \times t$

c) $X = a\sin\sqrt{\frac{g}{L}} \times t$

d) $X = a\cos\sqrt{\frac{g}{L}} \times t$

353. The displacement of a particle along the x axis is given by $x = a\sin^2\omega t$. The motion of the particle corresponds to

a) Simple harmonic motion of frequency $\omega/2\pi$
b) Simple harmonic motion of frequency ω/π
c) Simple harmonic motion of frequency $3\omega/2\pi$
d) Non simple harmonic motion

354. The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then new time period will be

a) T
b) $T/2$
c) $2\ T$
d) $T/4$

355. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F\ (t)$ proportional to $\cos\omega t(\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to

a) $\frac{m}{\omega_0^2 - \omega^2}$
b) $\frac{1}{m(\omega_0^2 - \omega^2)}$
c) $\frac{1}{m(\omega_0^2 + \omega^2)}$
d) $\frac{m}{\omega_0^2 + \omega^2}$

356. Two pendulums of lengths 1m and 1.21m respectively start swinging together with same amplitude. The number of vibrations that will be executed by the longer pendulum before the two will swing together again are

a) 9
b) 10
c) 11
d) 12

357. A body is vibrating in simple harmonic motion with an amplitude of $0.06\ m$ and frequency of $15\ Hz$. The velocity and acceleration of body is

a) $5.65\ m/s$ and $5.32 \times 10^2 m/s^2$
b) $6.82\ m/s$ and $7.62 \times 10^2 m/s^2$

c) $8.91\ m/s$ and $8.21 \times 10^2 m/s^2$ d) $9.82\ m/s$ and $9.03 \times 10^2 m/s^2$

358. If a simple pendulum of length l has maximum angular displacement θ, then the maximum kinetic energy of bob of mass m is

  a) $\dfrac{1}{2} \times \left(\dfrac{l}{g}\right)$ b) $\dfrac{1}{2} \times \dfrac{mg}{l}$

  c) $mgl \times (1 - \cos\theta)$ d) $\dfrac{1}{2} \times mgl \sin\theta$

359. The time period of a simple pendulum in a lift descending with constant acceleration g is

               c) Zero   d) Infinite

  a) $T = 2\pi\sqrt{\dfrac{l}{g}}$ b) $T = 2\pi\sqrt{\dfrac{l}{2g}}$

360. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be
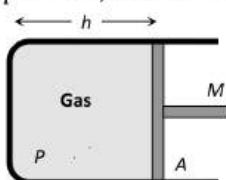
  a) $T = 2\pi\sqrt{\left(\dfrac{Mh}{PA}\right)}$ b) $T = 2\pi\sqrt{\left(\dfrac{MA}{Ph}\right)}$ c) $T = 2\pi\sqrt{\left(\dfrac{M}{PAh}\right)}$ d) $T = 2\pi\sqrt{MPhA}$

361. A particle starts SHM form the mean position. Its amplitude is a and total energy E. At one instant its kinetic energy is $3E/4$ its displacement at this instant is

  a) $y = a/\sqrt{2}$ b) $y = \dfrac{a}{2}$ c) $y = \dfrac{a}{\sqrt{3/2}}$ d) $y = a$

362. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing S.H.M. is

  a) $-a$ b) $+a$ c) $\pm a$ d) $\pm a/4$

363. The total energy of the body executing S.H.M. is E. Then the kinetic energy when the displacement is half of the amplitude, is

  a) $\dfrac{E}{2}$ b) $\dfrac{E}{4}$ c) $\dfrac{3E}{4}$ d) $\dfrac{\sqrt{3}}{4}E$

364. When the displacement is half of the amplitude, then when fraction of the total energy of a simple harmonic oscillator is kinetic?

  a) 2/7th   b) 3/4th   c) 2/9th   d) 5/7th

365. A simple pendulum has a length l and the mass of the bob is m. The bob is given a charge q coulomb. The pendulum is suspended between the vertical plates of a charged parallel plate capacitor. If E is the electric field strength between the plates, the time period of the pendulum is given by

  a) $2\pi\sqrt{\dfrac{l}{g}}$ b) $2\pi\sqrt{\dfrac{l}{\sqrt{g + \dfrac{qE}{m}}}}$ c) $2\pi\sqrt{\dfrac{l}{\sqrt{g - \dfrac{qE}{m}}}}$ d) $2\pi\sqrt{\dfrac{l}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}}$

366. A particle of mass m is executing oscillations about the origin on the x-axis with amplitude A. Its PE is given as $U_{(x)} = \alpha x^4$, where α is positive constant. The x- coordinate of mass where potential energy is one-third of the KE of particle, is

  a) $\pm\dfrac{A}{\sqrt{3}}$ b) $\pm\dfrac{A}{\sqrt{2}}$ c) $\pm\dfrac{A}{3}$ d) $\pm\dfrac{A}{2}$

367. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be
    a) $T\sqrt{n}$
    b) $T/\sqrt{n}$
    c) nT
    d) T

368. The potential energy of a particle with displacement X is $U(X)$. The motion is simple harmonic, when (K is a positive constant)
    a) $U = \dfrac{KX^2}{2}$
    b) $U = KX^2$
    c) $U = K$
    d) $U = KX$

369. Two identical springs are connected to mass m as shown (k = spring constant). If the period of the configuration in (a) is 2 s, the period of the configuration in (b) is
    a) $2\sqrt{2}$ s
    b) 1 s
    c) $\dfrac{1}{\sqrt{2}}$ s
    d) $\sqrt{2}$ s

370. The bob of a simple pendulum is displaced from its equilibrium position O to a position Q which is at height h above O and the bob is then released. Assuming the mass of the bob to be m and time period of oscillations to be $2.0\ s$, the tension in the string when the bob passes through O is
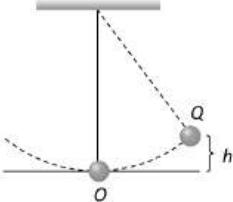
    a) $m(g + \pi\sqrt{2gh})$
    b) $m(g + \sqrt{\pi^2 gh})$
    c) $m\left(g + \sqrt{\dfrac{\pi^2}{2}gh}\right)$
    d) $m\left(g + \sqrt{\dfrac{\pi^2}{3}gh}\right)$

371. A mass m is suspended by means of two coiled spring which have the same length in unstretched condition as in figure. Their force constant are k_1 and k_2 respectively. When set into vertical vibrations, the period will be
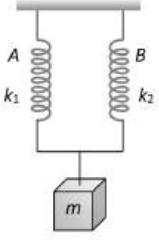
    a) $2\pi\sqrt{\left(\dfrac{m}{k_1 k_2}\right)}$
    b) $2\pi\sqrt{m\left(\dfrac{k_1}{k_2}\right)}$
    c) $2\pi\sqrt{\left(\dfrac{m}{k_1 - k_2}\right)}$
    d) $2\pi\sqrt{\left(\dfrac{m}{k_1 + k_2}\right)}$

372. A clock which keeps correct time at 20°C, is subjected to 40°C. If coefficient of linear expansion of the pendulum is $12 \times 10^{-6}/°C$. How much will it gain or loose in time
    a) $10.3\ seconds/day$
    b) $20.6\ seconds/day$
    c) $5\ seconds/day$
    d) $20\ minutes/day$

373. The motion of a particle executing SHM is given by $x = 0.01 \sin 100p(t + 0.05)$, where x is in metre and time t is in second. The time period is
    a) 0.2 s
    b) 0.1 s
    c) 0.02 s
    d) 0.01 s

374. A mass of $10\ kg$ is suspended from a spring balance. It is pulled aside by a horizontal string so that it makes angle of 60° with the vertical. The new reading of the balance is
    a) $10\sqrt{3}\ kg\ wt$
    b) $20\sqrt{3}kg\ wt$
    c) $20\ kg\ wt$
    d) $10\ kg\ wt$

375. The motion of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$
    a) The motion is oscillatory but not SHM
    b) The motion is SHM with amplitude a + b
    c) The motion is SHM with amplitude $a^2 + b^2$
    d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

376. A body is executing S.H.M. When its displacement from the mean position is $4\ cm$ and $5\ cm$, the corresponding velocity of the body is $10\ cm/s$ and $8\ cm/s$. Then the time period of the body is

a) $2\pi\ s$ b) $\pi/2\ s$ c) $\pi\ s$ d) $3\pi/2s$

377. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be
(If it is a second's pendulum on earth)

a) $\dfrac{1}{\sqrt{2}}s$ b) $2\sqrt{2}\ s$ c) $2\ s$ d) $\dfrac{1}{2}s$

378. The motion of a particle executing S.H.M. is given by $x = 0.01\sin 100\pi(t + .05)$, where x is in $metres$ and time is in seconds. The time period is

a) $0.01\ s$ b) $0.02\ s$ c) $0.1\ s$ d) $0.2\ s$

379. Let T_1 and T_2 be the time period of spring A and B when mass M is suspended from one end of each spring. If both springs are taken in series and the same mass M is suspended from the series combination, the time period is T, then

a) $T = T_1 + T_2$ b) $\dfrac{1}{T} = \dfrac{1}{T_1} + \dfrac{1}{T^2}$ c) $T^2 = T_1^2 + T_2^2$ d) $\dfrac{1}{T^2} = \dfrac{1}{T_1^2} + \dfrac{1}{T_2^2}$

380. A $1.00 \times 10^{-20}kg$ particle is vibrating with simple harmonic motion with a period of $1.00 \times 10^{-5}s$ and a maximum speed of $1.00 \times 10^3 m/s$. The maximum displacement of the particle is

a) $1.59\ mm$ b) $1.00\ m$ c) $10\ m$ d) None of these

381. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time and displacement x.
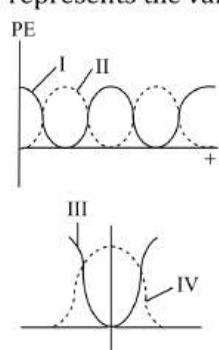
a) I,III b) II,III c) I,IV d) II,IV

382. Which of the following function represents a simple harmonic oscillation

a) $\sin\omega t - \cos\omega t$ b) $\sin^2\omega t$ c) $\sin\omega t + \sin 2\omega t$ d) $\sin\omega t - \sin 2\omega t$

383. In damped oscillations, the amplitude of oscillations is reduced to one-third of its initial value a_0 at the end of 100 oscillations. When the oscillator completes 200 oscillations, its amplitude must be

a) $a_0/2$ b) $a_0/4$ c) $a_0/6$ d) $a_0/9$

384. Two springs of force constant K and $2K$ are connected to a mass as shown below. The frequency of oscillation of the mass is
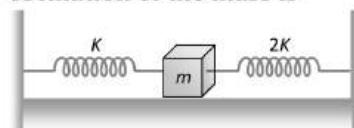
a) $(1/2\pi)\sqrt{(K/m)}$ b) $(1/2\pi)\sqrt{(2K/m)}$ c) $(1/2\pi)\sqrt{(3K/m)}$ d) $(1/2\pi)\sqrt{(m/K)}$

385. The amplitude of SHM y
$= 2(\sin 5\pi t + \sqrt{2}\cos\pi t)$ is

a) 2 b) $2\sqrt{2}$ c) 4 d) $2\sqrt{3}$

386. A particle doing simple harmonic motion, amplitude $= 4\ cm$, time period $= 12\ s$. The ratio between time taken by it in going from its mean position to $2\ cm$ and from $2\ cm$ to extreme position is

a) 1 b) 1/3 c) 1/4 d) 1/2

387. If the mass of an oscillator is numerically equal to its force constant, then the frequency is

a) π b) 2π c) $\dfrac{1}{\pi}$ d) $\dfrac{1}{2\pi}$

388. Two simple harmonic motions are represented by the equations

$y_1 = 0.1 \sin\left(100\,\pi t + \dfrac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$.

The phase difference of the velocity of particle 1, with respect to the velocity of particle 2 is

a) $\dfrac{-\pi}{6}$ b) $\dfrac{\pi}{3}$ c) $\dfrac{-\pi}{3}$ d) $\dfrac{\pi}{6}$

389. The potential energy of a particle (U_x) executing SHM is given by

a) $U_x = \dfrac{k}{2}(x-a)^2$ b) $U_x = k_1 x + k_2 x^2 + k_3 x^3$

c) $U_x = Ae^{-bx}$ d) $U_x = $ a constant

390. A spring has a certain mass suspended from it and its period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is now

a) $\dfrac{T}{2}$ b) $\dfrac{T}{\sqrt{2}}$ c) $\sqrt{2}T$ d) $2T$

391. A body performs S.H.M. its kinetic energy K varies with time t as indicated by graph
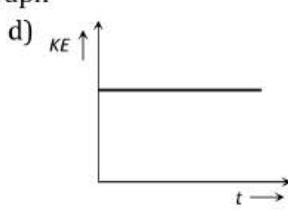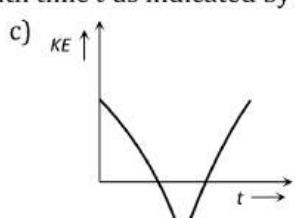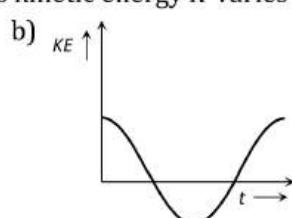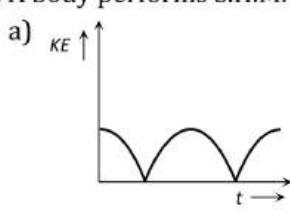
392. A particle is executing SHM with amplitude a. when the PE of a particle is one-fourth of its maximum value during the oscillation, its displacement from the equilibrium position will be

a) a/4   b) a/3   c) a/2   d) 2a/3

393. A particle free to move along the x-axis has potential energy given as

$U(x) = k[1 - \exp(-x^2)]$ $(\text{for} -\infty \le +\infty)$

Where k is a positive constant of appropriate dimensions. Then

a) At points away from origin, the particle is in equilibrium

b) For any finite non-zero value of x, there is a force directed away from the origin

c) Its total mechanical energy is $k/2$ and it is equal to its kinetic energy at origin

d) At $x = 0$, the motion of the particle is simple harmonic

394. The force constant of two springs are K_1 and K_2. Both are stretched till their elastic energies are equal. If the stretching forces are F_1 and F_2, then $F_1 : F_2$ is

a) $K_1 : K_2$ b) $K_2 : K_1$ c) $\sqrt{K_1} : \sqrt{K_2}$ d) $K_1^2 : K_2^2$

395. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes simple harmonic oscillations with a time period T. If the mass is increased by m then the time period becomes $\left(\dfrac{5}{4}T\right)$. The ratio of $\dfrac{m}{M}$ is

a) 9/16   b) 25/16   c) 4/5   d) 5/4

396. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes
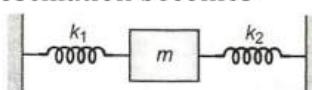
a) $f/2$ b) $f/4$ c) $4f$ d) $2f$

397. A simple pendulum performs simple harmonic motion about $X = 0$ with an amplitude A and time period T. The speed of the pendulum at $X = \frac{A}{2}$ will be

a) $\dfrac{\pi A\sqrt{3}}{T}$　　　　　b) $\dfrac{\pi A}{T}$　　　　　c) $\dfrac{\pi A\sqrt{3}}{2T}$　　　　　d) $\dfrac{3\pi^2 A}{T}$

398. A mass 1 kg suspended from a spring shoes force constant is $400\ \mathrm{Nm^{-1}}$, executes simple harmonic oscillation. When the total energy of the oscillator is 2j, the maximum acceleration experienced by the mass will be

a) $2\ \mathrm{ms^{-2}}$　　　　　b) $4\ \mathrm{ms^{-2}}$　　　　　c) $40\ \mathrm{ms^{-2}}$　　　　　d) $400\ \mathrm{ms^{-2}}$

399. Two mutually perpendicular simple harmonic vibrations have same amplitude, frequency and phase. When they superimpose, the resultant from of vibration will be

a) A circle　　　　　b) An ellipse　　　　　c) A straight line　　　　　d) A parabola

400. There is a simple pendulum hanging from the ceiling of a lift. When the lift is stand still, the time period of the pendulum is T. If the resultant acceleration becomes g/4, then the new time period of the pendulum is

a) 0.8 T　　　　　b) 0.25 T　　　　　c) 2 T　　　　　d) 4 T

401. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement x from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is

a) $(a^2 - x^2\omega^2)/x^2\omega^2$　　　　　　　　　b) $x^2\omega^2/(a^2 - x^2\omega^2)$

c) $(a^2 - x^2)/x$　　　　　　　　　d) $x^2/(a^2 - x^2)$

402. The equation of a damped simple harmonic motion is $m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0$. Then the angular frequency of oscillation is

a) $\omega = \left(\dfrac{k}{m} - \dfrac{b^2}{4m^2}\right)^{1/2}$　　b) $\omega = \left(\dfrac{k}{m} - \dfrac{b}{4m}\right)^{1/2}$　　c) $\omega = \left(\dfrac{k}{m} - \dfrac{b^2}{4m}\right)^{1/2}$　　d) $\omega = \left(\dfrac{k}{m} - \dfrac{b^2}{4m^2}\right)$

403. In a simple harmonic motion maximum velocity is at

a) Extreme position　　　　　　　　　b) Half of extreme position

c) Equilibrium position　　　　　　　　　d) Between extreme and equilibrium position

404. The displacement-time graph of a particle executing SHM is as shown in the figure.
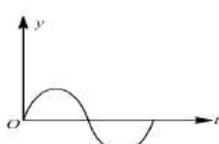
The corresponding force-time graph of the particle is

a)　　　　　　　b)　　　　　　　c)　　　　　　　d)
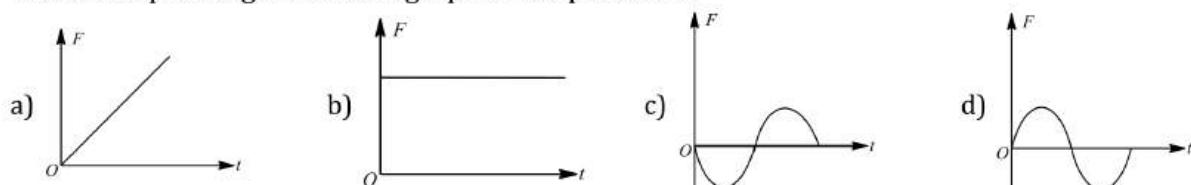
405. When the amplitude of a body executing SHM become twice what happens?

a) Maximum potential energy is doubled　　　　　b) Maximum kinetic energy is doubled

c) Total energy is doubled　　　　　d) Maximum velocity is doubled

406. A spring with 10 coils has spring constant k. It is exactly cut into two halves, then each of these new springs will have a spring constant

a) $k/2$　　　　　b) $3k/2$　　　　　c) $2k$　　　　　d) $3k$

407. A simple pendulum of length l has a bob of mass m, with a charge q on it. A vertical sheet of change, with surface charge density σ passes through the point of suspension. At equilibrium, the spring makes an angle θ with the vertical. Its time period of oscillations is T in this position. Then

a) $\tan \theta = \dfrac{\sigma q}{2\varepsilon_0 mg}$ b) $\tan \theta = \dfrac{\sigma q}{\varepsilon_0 mg}$ c) $T > 2\pi \sqrt{\dfrac{1}{g}}$ d) $T = 2\pi \sqrt{\dfrac{1}{g}}$

408. The amplitude of damped oscillator becomes $\frac{1}{2}$ in 2 s.
Its amplitude after 6 is $1/n$ times the original. Then n is equal to
a) 2^3 b) 3^2 c) $3^{\frac{1}{3}}$ d) 3^3

409. A particle executes harmonic motion with an angular velocity and maximum acceleration of 3.5 rad/s and 7.5 m/s^2 respectively. The amplitude of oscillation is
a) $0.28\ m$ b) $0.36\ m$ c) $0.53\ m$ d) $0.61\ m$

410. A simple harmonic wave having an amplitude A and time period T is represented by the equation $y = 5 \sin \pi(t + 4)$m. Then the value of A in (metre) and T in (second) are
a) $A = 10, T = 2$ b) $A = 5, T = 1$ c) $A = 10, T = 1$ d) $A = 5, T = 2$

411. The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of amplitude. The total energy of the particle be
a) $18\ J$ b) $10\ J$ c) $12\ J$ d) $2.5\ J$

412. Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Then the angular frequency of oscillation of m_2 is
a) $\sqrt{k/m_1}$
b) $\sqrt{k/m_2}$
c) $\sqrt{k/(m_1 + m_2)}$
d) $\sqrt{k/(m_1 - m_2)}$

413. The kinetic energy of a particle executing S.H.M. is 16 J when it is in its mean position. If the amplitude of oscillations is 25 cm and the mass of the particle is 5.12 kg, the time period of its oscillation is
a) $\dfrac{\pi}{5} s$ b) $2\pi\ s$ c) $20\pi\ s$ d) $5\pi\ s$

414. Average value of KE and PE over entire time period is
a) $0, \dfrac{1}{2}m\,\omega^2 A^2$
b) $\dfrac{1}{2}m\,\omega^2 A^2, 0$
c) $\dfrac{1}{2}m\,\omega^2 A^2, \dfrac{1}{2}m\,\omega^2 A^2$
d) $\dfrac{1}{4}m\,\omega^2 A^2, \dfrac{1}{4}m\,\omega^2 A^2$

415. A particle has simple harmonic motion. The equation of its motion is $x = 5 \sin\left(4t - \dfrac{\pi}{6}\right)$, where x is its displacement. If the displacement of the particle is 3 units, then it velocity is
a) $\dfrac{2\pi}{3}$ b) $\dfrac{5\pi}{6}$ c) 20 d) 16

416. This time period of a particle undergoing SHM is 16 s. It starts motion from the mean position. After 2 s, its velocity is 0.4 ms^{-1}. The amplitude is
a) 1.44 m b) 0.72 m c) 2.88 m d) 0.36 m

417. The maximum velocity of a particle executing SHM is V. If the amplitude is doubled and the time period of oscillation decreased to 1/3 of its original value, the maximum velocity becomes
a) $18\ V$ b) $12\ V$ c) $6\ V$ d) $3\ V$

418. The equation of S.H.M. is $y = a \sin(2\pi nt + \alpha)$, then its phase at time t is
a) $2\pi nt$ b) α c) $2\pi nt + \alpha$ d) $2\pi t$

419. A spring executes SHM with mass of $10kg$ attached to it. The force constant of spring is 10N/m. If at any instant its velocity is 410cm/s, the displacement will be (where amplitude is 0.5m)
a) 0.09 m b) 0.3 m c) 0.03 m d) 0.9 m

420. The acceleration d^2x/dt^2 of a particle varies with displacement x as $\dfrac{d^2x}{dt^2} = -kx$ where k is a constant of the motion. The time period T of the motion is equal to
a) $2\pi k$ b) $2\pi\sqrt{k}$ c) $2\pi /\sqrt{k}$ d) $2\pi/k$

421. Lissajous figure shown in figure. Corresponds to which one of the following?

a) Phase difference $\pi/2$ and period $1:2$ b) Phase difference $3\pi/4$ and period $1:2$

c) Phase difference $\pi/4$ and period $2:1$ d) Phase difference $2\pi/3$ and period $2:1$

422. Mark the wrong statement

a) All S.H.M.'s have fixed time period

b) All motions having same time period are S.H.M.

c) In S.H.M. total energy is proportional to square of amplitude

d) Phase constant of S.H.M. depends upon initial conditions

423. The displacement equation of a particle is $x = 3\sin 2t + 4\cos 2t$. The amplitude and maximum velocity will be respectively

a) 5, 10   b) 3, 2   c) 4, 2   d) 3, 4

424. Which of the following combination of Lissajous' figure will be like eight (8)?

a) $x = a\sin 4\omega t, y = b\sin \omega t$ b) $x = a\sin 2\omega t, y = b\sin \omega t$

c) $x = a\sin 2\omega t, y = b\sin 2\omega t$ d) $x = a\sin \omega t, y = b\sin 4\omega t$

425. A horizontal plank has a rectangular block placed on it. The plank stars oscillating vertically and simple harmonically with an amplitude of 40 cm. the block just loses contact with the plank when the later is momentary at rest. Then

a) the period of oscillating is $2\pi/5$ s

b) The block weighs double its weight when the plank is at one of the positions of momentary at rest.

c) the block weight 1.5 times its weight on the plank half way down

d) The block weights its true weight on the plank, when the latter moves fastest

426. A simple pendulum is vibrating in an evacuated chamber, it will oscillate with

a) Increasing amplitude   b) Constant amplitude   c) Decreasing amplitude   d) First (c) then (a)

427. When the displacement is half the amplitude, the ratio of potential energy to the total energy is

a) $\dfrac{1}{2}$ b) $\dfrac{1}{4}$ c) 1   d) $\dfrac{1}{8}$

428. A S.H.M. is represented by $x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of the S.H.M. is

a) 10 cm b) 20 cm c) $5\sqrt{2} cm$ d) 50 cm

429. If a body is executing simple harmonic motion then

a) At extreme positions, the total energy is zero

b) At equilibrium position, the total energy is in the form of potential energy

c) At equilibrium position, the total energy is in the form of kinetic energy

d) At extreme position, the total energy is infinite

430. Two simple harmonic motions are represented by

$$y_1 = 5\left[\sin 2\pi t + \sqrt{3}\cos 2\pi t\right]$$

and $y_2 = 5\sin\left(2\pi t + \dfrac{\pi}{4}\right)$

The ratio of their amplitudes is

a) $1:1$ b) $2:1$ c) $1:3$ d) $\sqrt{3}:1$

431. The equation of a simple harmonic wave is given by $y = 5\sin\dfrac{\pi}{2}(100t - x)$, where x and y are in metre and time is in second. The period of the wave in second will be

a) 0.04   b) 0.01   c) 1   d) 5

432. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

a) 11% b) 21% c) 42% d) 10.5%

433. A body is executing Simple Harmonic Motion. At a displacement x its potential energy is E_1 and at a displacement y its potential energy is E_2. The potential energy E at displacement $(x + y)$ is

a) $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$ b) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$ c) $E = E_1 - E_2$ d) $E = E_1 + E_2$

434. The bob of a simple pendulum of mass m and total energy E will have maximum linear momentum equal to

a) $\sqrt{\dfrac{2E}{m}}$ b) $\sqrt{2mE}$ c) $2mE$ d) mE^2

435. A particle of mass m executes simple harmonic motion with amplitude a and frequency v. The average kinetic energy during its motion from the position of equilibrium to the end is

a) $\pi^2 m a^2 v^2$ b) $\dfrac{1}{4} m a^2 v^2$ c) $4\pi^2 m a^2 v^2$ d) $2\pi^2 m a^2\, v^2$

436. Two blocks with masses $m_1 = 1kg$ and $m_2 = 2$ kg are connected by a spring constant $k = 24\ Nm^{-1}$ and placed on a frictionless horizontal surface. The block m_1 is imparted an initial velocity $v_0 = 12$ cms $^{-1}$ to the right, the amplitude of oscillation is

a) 1 cm b) 2 cm c) 3 cm d) 4 cm

437. The amplitude of a damped oscillator becomes half in one minute. The amplitude after 3 $minute$ will be $\dfrac{1}{X}$ times the original, where X is

a) 2×3 b) 2^3 c) 3^2 d) 3×2^2

438. A pendulum bob of mass m is hanging from a fixed point by a light thread of length l. A horizontal speed v_0 is imparted to the bob so that it takes up horizontal position. If g is the acceleration due to gravity, then v_0 is

a) mgl b) $\sqrt{2gl}$ c) $\sqrt{gl}$ d) gl

439. A particle is executing simple harmonic motion with an amplitude A and time period T. The displacement of the particle after $2T$ period from its initial position is

a) A b) $4A$ c) $8A$ d) Zero

440. The time period of a simple pendulum is T. When the length is increased by 10 cm, its period is T_1. When the length is decreased by 10 cm, its period is T_2. Then, relation between T, T_1 and T_2 is

a) $\dfrac{2}{T^2} = \dfrac{1}{T_1^2} + \dfrac{1}{T_2^2}$ b) $\dfrac{2}{T^2} = \dfrac{1}{T_1^2} - \dfrac{1}{T_2^2}$ c) $2T^2 = T_1^2 + T_2^2$ d) $2T^2 = T_1^2 - T_2^2$

441. A simple pendulum is taken from the equator to the pole. Its period

a) Decreases b) Increases c) Remains the same d) Decreases and then increases

442. A spring of spring constant k is cut into two equal parts. A block of mass m is attached with one part of spring. What is the frequency of the system if α is frequency of block with original spring?

a) $\sqrt{2}\,\alpha$ b) $\alpha/2$ c) 2α d) α

443. If a spring extends by x on loading then the energy stored in the spring is (if T is the tension and k is the force constant of the of the spring).

a) $\dfrac{T^2}{2x}$ b) $\dfrac{T^2}{2k}$ c) $\dfrac{2k}{T^2}$ d) $\dfrac{2T^2}{k}$

444. The SHM of a particle is given by

$$x(t) = 5 \cos\left(2\pi t + \frac{\pi}{4}\right) \text{ in MKS units.}$$

Calculate the displacement and the magnitude of acceleration of the particle at $t = 1.5$ s.

a) -3.0 m, $100\ m/s^2$ b) $+2.54$ m, $200\ m/s^2$ c) -3.54 m, $140\ m/s^2$ d) $+3.55$ m, $120\ m/s^2$

445. Two identical springs of constant K are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be
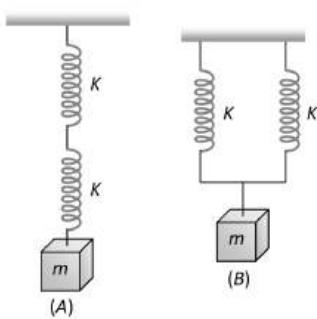
a) 2 :1   b) 1 :1   c) 1 :2   d) 4 :1

446. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a change +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then SHM of the block will be
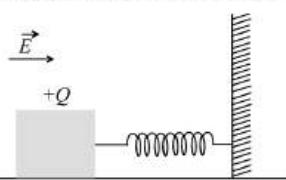

a) Of the same frequency and with shifted mean position
b) Of the same frequency and with the same mean position
c) Of changed frequency and with shifted mean position
d) Of changed frequency and with the same mean position

447. The acceleration due to gravity at a place is $\pi^2 \ m/s^2$. Then the time period of a simple pendulum of length one *metre* is

a) $\dfrac{2}{\pi} s$ b) $2\pi \ s$ c) $2 \ s$ d) $\pi \ s$

448. Three masses $700g$, $500g$, and $400g$ are suspended at the end of a spring a shown and are in equilibrium. When the $700g$ mass is removed, the system oscillates with a period of 3 seconds, when the $500g$ mass is also removed, it will oscillate with a period of
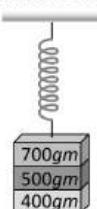
a) $1 \ s$ b) $2 \ s$ c) $3 \ s$ d) $\sqrt{\dfrac{12}{5}} s$

449. When a body of mass 1.0 kg is suspended from a certain light spring hanging vertically, its length increases by 5 cm. By suspending 2.0 kg block to the spring and if the block is pulled through 10 cm and released, the maximum velocity of it in ms^{-1} is (g $= 10$ ms^{-2})

a) 0.5   b) 1   c) 2   d) 4

450. The velocity of simple pendulum is maximum at

a) Extremes   b) Half displacement   c) Mean position   d) Every where

451. Two equal negative charge –q are fixed points (0, a) and (0,-0) on the Y-axis. A positive charge Q is released from rest at point (2a, 0) on the X-axis. The charge Q will

a) execute SHM about origin   b) move to infinity
c) Move to the origin and remained at rest   d) execute oscillatory but not SHM

452. A simple pendulum, suspended from the ceiling of a stationary van, has time period T. If the van starts moving with a uniform velocity the period of the pendulum will be

a) Less than T b) Equal to $2T$ c) Greater than T d) Unchanged

453. A particle is executing SHM with amplitude a. When the PE of a particle is one-fourth of its maximum value during the oscillation, its displacement from the equilibrium position will be

a) $a/4$ b) $a/3$ c) $a/2$ d) $2a/3$

454. A mass $m = 100\ g$ is attached at the end of a light spring which oscillates on a frictionless horizontal table with an amplitude equal to $0.16\ metre$ and time period equal to $2\ sec$. Initially the mass is released from rest at $t = 0$ and displacement $x = -0.16\ metre$. The expression for the displacement of the mass at any time t is

a) $x = 0.16\cos(\pi t)$ b) $x = -0.16\cos(\pi t)$
c) $x = 0.16\sin(\pi t + \pi)$ d) $x = -0.16\sin(\pi t + \pi)$

455. Acceleration of a particle, executing SHM, at it's mean position is

a) Infinity   b) Varies   c) Maximum   d) Zero

456. The equation of SHM is given by

$x = 3\sin 20\pi t + 4\cos 20\pi t$

Where x is in cm and t is I second. The amplitude is

a) 7 cm   b) 4 cm   c) 5 cm   d) 3 cm

457. A S.H.M. has amplitude $'a'$ and time period T. The maximum velocity will be

a) $\dfrac{4a}{T}$ b) $\dfrac{2a}{T}$ c) $2\pi\sqrt{\dfrac{a}{T}}$ d) $\dfrac{2\pi a}{T}$

458. The magnitude of maximum acceleration is π times that of maximum velocity of a simple harmonic oscillator. The time period of the oscillator in second is

a) 4   b) 2   c) 1   d) 0.5

459. If two springs A and B with spring constants $2\ k$ and k, are stretched separately by same suspended weight, then the ratio between the work done in stretched A and B is

a) 1:2   b) 1:4   c) 1:3   d) 4:1

460. An electric motor of mass $40\ kg$ is mounted on four vertical springs each having constant at $4000\ Nm^{-1}$. The period with which the motor vibrates vertically is

a) $0.314\ s$ b) $3.14\ s$ c) $0.628\ s$ d) $0.56\ s$

461. When the kinetic energy of a body executing S.H.M. is $1/3$ of the potential energy. The displacement of the body is x percent of the amplitude, where x is

a) 33   b) 87   c) 67   d) 50

462. A mass m is suspended separately by two different springs in successive order then a time period is t_1 and t_2 respectively. If m is connected by both spring as shown in figure, then time period is t_0, the correct relation is
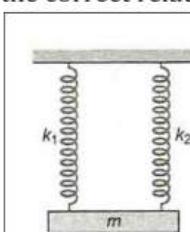
a) $t_0^2 = t_1^2 + t_2^2$ b) $t_0^{-2} = t_1^{-2} + t_2^{-2}$
c) $t_0^{-1} = t_1^{-1} + t_2^{-1}$ d) $t_0 = t_1 + t_2$

463. Two blocks each of mass m are connected to a spring of spring constant k. If both are given velocity v in opposite directions, then the maximum elongation of the spring is
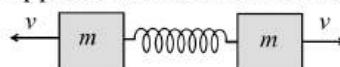
a) $\sqrt{\dfrac{mv^2}{k}}$ b) $\sqrt{\dfrac{2mv^2}{k}}$ c) $\sqrt{\dfrac{mv^2}{2k}}$ d) $2\sqrt{\dfrac{mv^2}{k}}$

464. A particle of mass 10 g is describing S.H.M. along a straight line with period of 2 s and amplitude of 10 cm. Its kinetic energy when it is at 5 cm from its equilibrium position is

a) $37.5\pi^2 ergs$ b) $3.75\pi^2 ergs$ c) $375\pi^2 ergs$ d) $0.375\pi^2 ergs$

465. The graph shows the variation of displacement of a particle executing S.H.M. with time. We infer from this graph that
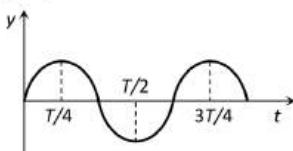
a) The force is zero at time $3T/4$ b) The velocity is maximum at time $T/2$

c) The acceleration is maximum at time T d) The P.E. is equal to total energy at time $T/2$

466. A point particle of mass 0.1 kg is executing SHM of amplitude 0.1m. When the particle passes through the mean position, its KE is 8×10^{-3}J. The equation of motion of this particle, if its initial phase of oscillation is 45^0 is

a) $y = 0.1\sin\left(\frac{r}{4} + \frac{\pi}{4}\right)$ b) $y = 0.1\sin\left(\frac{t}{2} + \frac{\pi}{4}\right)$

c) $y = 0.1\sin\left(4t - \frac{\pi}{4}\right)$ d) $y = 0.1\sin\left(4t + \frac{\pi}{4}\right)$

467. A man weighing 60 kg stands on the horizontal platform of a spring balance. The platform starts executing simple harmonic motion of amplitude 0.1 m and frequency $\frac{2}{\pi} Hz$. Which of the following statements is correct
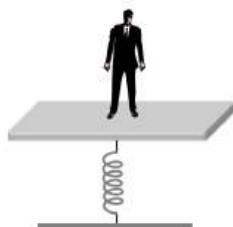
a) The spring balance reads the weight of man as 60 kg

b) The spring balance reading fluctuates between 60 kg and 70 kg

c) The spring balance reading fluctuates between 50 kg and 60 kg

d) The spring balance reading fluctuates between 50 kg and 70 kg

468. The period of a simple pendulum, whose bob is hollow metallic sphere, is T. The period is T_1 when the bob is filled with sand, T_2 when it is filled with mercury and T_3 when it is half filled with mercury. Which of the following is true
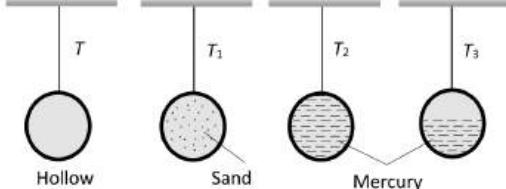
a) $T = T_1 = T_2 > T_3$ b) $T_1 = T_2 = T_3 > T$ c) $T > T_3 > T_1 = T_2$ d) $T = T_1 = T_2 < T_3$

469. Two simple harmonic motions of angular frequency 100 and 1000 rad/s have the same displacement amplitude. The ratio of their maximum acceleration is

a) $1:10$ b) $1:10^2$ c) $1:10^3$ d) $1:10^4$

470. A chimpanzee swinging on a swing in a sitting position, stands up suddenly, the time period will

a) Become infinite   b) Remain same   c) Increase   d) Decrease

471. Ratio of kinetic energy at mean position to potential energy at $A/2$ of a particle performing SHM

a) 2:1   b) 4:1   c) 8:1   d) 1:1

472. The motion of a particle varies with time according to the relation $y = a(\sin \omega t + \cos \omega t)$.
    a) The motion is oscillatory but not SHM   b) The motion is SHM with amplitude a
    c) The motion is SHM with amplitude $a\sqrt{2}$ d) The motion is SHM with amplitude $2a$

473. A particle, with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force.
$F = F_0 \sin \omega t$
If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle is maximum for $\omega = \omega_2$ then
    a) $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$ b) $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$
    c) $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$ d) $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$

474. A particle executing S.H.M. of amplitude $4\ cm$ and $T = 4\ s$. The time taken by it to move from positive extreme position to half the amplitude is
    a) $1\ s$ b) $1/3\ s$ c) $2/3\ s$ d) $\sqrt{3/2}\ s$

475. A heavy sphere of mass m is suspended by string of length l. The spear is made to revolve above a vertical line passing through the point of suspension in a horizontal circle such that the string always remains inclined to the vertical at $\angle\theta$. What is its period of revolution?
    a) $T = 2\pi\sqrt{\dfrac{l}{g}}$ b) $T = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$
    c) $T = 2\pi\sqrt{\dfrac{l\sin\theta}{g}}$ d) $T = 2\pi\sqrt{\dfrac{l\tan\theta}{g}}$

476. The maximum velocity and the maximum acceleration of a body moving in a simple harmonic oscillator are $2\ m/s$ and $4\ m/s^2$. Then angular velocity will be
    a) $3\ rad/s$ b) $0.5\ rad/s$ c) $1\ rad/s$ d) $2\ rad/s$

477. The vertical extension in a light spring by a weight of $1\ kg$ suspended from the wire is $9.8\ cm$. The period of oscillation
    a) $20\pi s$ b) $2\pi s$ c) $2\pi/10 s$ d) $200\pi s$

478. What is the velocity of the bob of a simple pendulum at its mean position, if it is able to rise to vertical height of 10 cm? (g=9.8 ms^{-2})
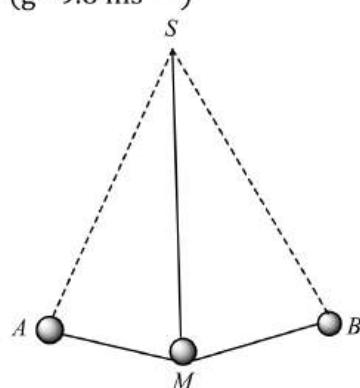
    a) $2.2\ \text{ms}^{-1}$ b) $1.8\ \text{ms}^{-1}$ c) $1.4\ \text{ms}^{-1}$ d) $0.6\ \text{ms}^{-1}$

479. A second's pendulum is placed in a space laboratory orbiting around the earth at a height $3R$, where R is the radius of the earth. The time period of the pendulum is
    a) Zero   b) $2\sqrt{3}\ s$ c) $4\ s$ d) Infinite

480. Velocity at mean position of a particle S.H.M. is v, they velocity of the particle at a distance equal to half of the amplitude

a) $4v$ b) $2v$ c) $\dfrac{\sqrt{3}}{2}v$ d) $\dfrac{\sqrt{3}}{4}v$

481. A point performs simple harmonic oscillation of period T and the equation of motion is given by $x = a\sin(w + \pi/6)$. After the elapse of what fraction of the time period the velocity of the point will be equal to half its maximum velocity

a) $\dfrac{T}{3}$ b) $\dfrac{T}{12}$ c) $\dfrac{T}{8}$ d) $\dfrac{T}{6}$

482. If a body is released into a tunnel dug across the diameter of earth, it executes simple harmonic motion with time period

a) $T = 2\pi\sqrt{\dfrac{R_e}{g}}$ b) $T = 2\pi\sqrt{\dfrac{2R_e}{g}}$ c) $T = 2\pi\sqrt{\dfrac{R_e}{2g}}$ d) $T = 2$ seconds

483. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is

a) $0.5\,\pi$ b) π c) 0.707π d) Zero

484. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is

a) $\sqrt{k_1/k_2}$ b) k_1/k_2 c) $\sqrt{k_2/k_1}$ d) k_2/k_1

485. Which one of the following equations does not represent SHM, $x = $ displacement and t = time. Parameters a, b and c are the constants of motion?

a) $x = a\sin bt$ b) $x = a\cos bt + c$

c) $x = a\sin bt + c\cos bt$ d) $x = a\sec bt + c\,\mathrm{cosec}\,bt$

486. An instantaneous displacement of a simple harmonic oscillator is $x = a\cos(\omega t\,\pi/4)$. Its speed will be maximum at time

a) $\pi/4\omega$ b) $\pi/2\omega$ c) π/ω d) $2\pi/\omega$

487. Two simple harmonic motions act on a particle. These harmonic motions are
$x = A\cos(\omega t + \delta);\ A\cos(\omega t + \alpha)$ when

a) an ellipse and the actual motion is counter clockwise   b) an ellipse and the actual motion is clockwise

c) a circle and the actual motion is counter clockwise   d) a circle and the actual motion is clockwise

488. Two springs have spring constants K_A and K_B and $K_A > K_B$. The work required to stretch them by same extension will be

a) More in spring A b) More in spring B c) Equal in both   d) Nothing can be said

489. The acceleration of a particle in S.H.M. is

a) Always zero   b) Always constant

c) Maximum at the extreme position   d) Maximum at the equilibrium position

490. A particle executing simple harmonic motion of amplitude 5 cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is

a) 3 Hz   b) 2 Hz   c) 4 Hz   d) 1 Hz

491. A particle is executing the motion $x = A\cos(\omega t - \theta)$. The maximum velocity of the particle is

a) $A\omega\cos\theta$ b) $A\omega$ c) $A\omega\sin\theta$ d) None of these

492. An ideal spring with spring constant $K = 200\ N/m$ is fixed on one end on a wall. If the spring is pulled with a force 10 N at the other end along its length, how much it will extended?

a) $5\ cm$ b) $2\ m$ c) $2\ cm$ d) $5\ m$

493. Two equations of two S.H.M. are $y = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha)$. The phase difference between the two is

a) $0°$ b) $\alpha°$ c) $90°$ d) $180°$

494. The total energy of a particle executing SHM is 80 J. What is the potential energy when the particle is at a distance of 3/4 of amplitude from the mean position?
a) 60 J
b) 10 J
c) 40 J
d) 45 J

495. An elastic string has a length l when tension in it is 5 N. Its length is h when tension is of 4 N. on subjecting the string to a tension of 9 N, its length will be
a) $l + h$
b) $l - h$
c) $(5 l - 4 h)$
d) $(l + h)/(h - l)$

496. A particle executes simple harmonic motion with a time period of 16 s. At time $t = 2$ s, the particle crosses the mean position while at $t = 4$ s, velocity is 4 ms^{-1}. The amplitude of motion in metre is
a) $\sqrt{2}\pi$
b) $16\sqrt{2}\pi$
c) $24\sqrt{2}\pi$
d) $\dfrac{32\sqrt{2}}{\pi}$

497. One-fourth length of a spring of force constant K is cut away. The force constant of the remaining spring will be
a) $\dfrac{3}{4}K$
b) $\dfrac{4}{3}K$
c) K
d) $4 K$

498. A block is resting on a piston which is moving vertically with SHM of period 1.0 s. At what amplitude of motion will the block and piston separate?
a) 0.2 m
b) 0.25 m
c) 0.3 m
d) 0.35 m

499. The kinetic energy and potential energy of a particle executing simple harmonic motion will be equal, when displacement
a) $\dfrac{a}{2}$
b) $a\sqrt{2}$
c) $\dfrac{a}{\sqrt{2}}$
d) $\dfrac{a\sqrt{2}}{3}$

500. A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity $4ms^{-1}$. It collides with a horizontal spring of force constant $200Nm^{-1}$. The maximum compression produced in the spring will be
a) 0.5 m
b) 0.6 m
c) 0.7 m
d) 0.2 m

501. A particle executes S.H.M. with a period of 6 second and amplitude of 3 cm. Its maximum speed in cm/s is
a) $\pi/2$
b) π
c) 2π
d) 3π

502. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is
a) $f/2$
b) f
c) $2f$
d) $4f$

503. A large horizontal surface moves up and down in SHM with an amplitude of 1 cm. if a mass of 10 kg (which is placed on the surface) id to remain continuously is in contact with it. The maximum frequency of SHM will be
a) 5 Hz
b) 0.5 Hz
c) 1.5 Hz
d) 10 Hz

504. The circular motion of a particle with constant speed is
a) Simple harmonic but not periodic
b) Periodic and simple harmonic
c) Neither periodic nor simple harmonic
d) Periodic but not simple harmonic

505. Masses m and 3 m are attached to the two ends of a spring of constant k. If the system vibrates freely. The period of oscillation will be
a) $\pi\sqrt{\dfrac{m}{k}}$
b) $2\pi\sqrt{\dfrac{m}{k}}$
c) $\pi\sqrt{\dfrac{3m}{k}}$
d) $2\pi\sqrt{\dfrac{3m}{k}}$

506. The equation of a simple harmonic wave is given by $y = 6 \sin 2\pi(2t - 0.1x)$, where x and y are in mm and t is in seconds. The phase difference between two particles 2 mm apart at any instant is
a) 54°
b) 72°
c) 18°
d) 36°

507. Two simple harmonic motion are represented by
$y_1 = 5(\sin 2\pi t + \sqrt{3} \cos 2\pi t)$
$y_2 = 5 \sin\left(2\pi t + \dfrac{\pi}{4}\right)$
The ratio of the amplitudes of two SHM's is

a) 1 :1   b) 1 :2   c) 2 :1   d) $1 : \sqrt{3}$

508. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\dfrac{A_1}{A_2}\right)$ is

     a) $\dfrac{M+m}{M}$ b) $\left(\dfrac{M}{M+m}\right)^{1/2}$ c) $\left(\dfrac{M+m}{M}\right)^{1/2}$ d) $\dfrac{M}{M+m}$

509. Two springs of constant k_1 and k_2 are joined in series. The effective spring constant of the combination is given by

     a) $\sqrt{k_1 k_2}$ b) $(k_1 + k_2)/2$ c) $k_1 + k_2$ d) $k_1 k_2/(k_1 + k_2)$

510. A particle starts oscillating simple harmonically from its equilibrium position with time period T. The ratio of KE and PE of other particle at the $t = T/12$ is

     a) 1:4   b) 2:1   c) 3:1   d) 4:1

OSCILLATIONS

: ANSWER KEY :

1)	c	2)	c	3)	b	4)	a	161)	d	162)	a	163)	b	164)	d
5)	c	6)	d	7)	b	8)	d	165)	b	166)	b	167)	c	168)	a
9)	b	10)	b	11)	b	12)	a	169)	c	170)	d	171)	b	172)	d
13)	a	14)	c	15)	a	16)	d	173)	d	174)	b	175)	d	176)	a
17)	a	18)	d	19)	d	20)	d	177)	d	178)	a	179)	d	180)	d
21)	b	22)	b	23)	c	24)	a	181)	c	182)	a	183)	d	184)	b
25)	b	26)	d	27)	d	28)	d	185)	c	186)	a	187)	d	188)	d
29)	c	30)	a	31)	b	32)	a	189)	c	190)	b	191)	a	192)	c
33)	d	34)	a	35)	c	36)	d	193)	a	194)	b	195)	b	196)	b
37)	b	38)	c	39)	b	40)	b	197)	b	198)	d	199)	d	200)	a
41)	c	42)	b	43)	c	44)	a	201)	c	202)	a	203)	b	204)	b
45)	c	46)	c	47)	d	48)	c	205)	b	206)	c	207)	d	208)	d
49)	c	50)	c	51)	a	52)	d	209)	b	210)	b	211)	a	212)	c
53)	c	54)	b	55)	b	56)	d	213)	c	214)	b	215)	a	216)	a
57)	b	58)	c	59)	c	60)	c	217)	a	218)	b	219)	b	220)	c
61)	a	62)	d	63)	a	64)	d	221)	a	222)	d	223)	b	224)	d
65)	c	66)	d	67)	d	68)	d	225)	c	226)	b	227)	b	228)	b
69)	d	70)	a	71)	a	72)	b	229)	d	230)	a	231)	b	232)	d
73)	c	74)	b	75)	d	76)	a	233)	b	234)	d	235)	b	236)	c
77)	a	78)	c	79)	c	80)	a	237)	a	238)	a	239)	b	240)	a
81)	c	82)	c	83)	c	84)	c	241)	b	242)	d	243)	c	244)	c
85)	c	86)	a	87)	a	88)	b	245)	d	246)	c	247)	a	248)	a
89)	b	90)	a	91)	c	92)	a	249)	b	250)	d	251)	a	252)	b
93)	a	94)	c	95)	b	96)	a	253)	c	254)	c	255)	b	256)	c
97)	b	98)	c	99)	d	100)	c	257)	c	258)	b	259)	c	260)	b
101)	b	102)	b	103)	b	104)	b	261)	a	262)	a	263)	b	264)	b
105)	d	106)	a	107)	b	108)	d	265)	b	266)	d	267)	c	268)	b
109)	a	110)	d	111)	b	112)	b	269)	b	270)	d	271)	c	272)	d
113)	d	114)	c	115)	a	116)	d	273)	c	274)	d	275)	a	276)	c
117)	c	118)	a	119)	c	120)	a	277)	c	278)	a	279)	d	280)	a
121)	c	122)	b	123)	a	124)	d	281)	b	282)	a	283)	a	284)	c
125)	a	126)	c	127)	b	128)	c	285)	d	286)	a	287)	a	288)	d
129)	d	130)	c	131)	d	132)	b	289)	d	290)	b	291)	c	292)	a
133)	a	134)	b	135)	b	136)	b	293)	c	294)	b	295)	c	296)	c
137)	c	138)	d	139)	a	140)	b	297)	a	298)	a	299)	c	300)	d
141)	a	142)	d	143)	c	144)	b	301)	a	302)	c	303)	c	304)	b
145)	d	146)	c	147)	b	148)	b	305)	a	306)	c	307)	c	308)	b
149)	d	150)	b	151)	d	152)	b	309)	b	310)	d	311)	c	312)	c
153)	c	154)	c	155)	b	156)	c	313)	b	314)	c	315)	c	316)	c
157)	c	158)	b	159)	a	160)	d	317)	a	318)	a	319)	b	320)	d

321) a	322) b	323) b	324) b	421) a	422) b	423) a	424) b
325) d	326) c	327) b	328) b	425) b	426) b	427) b	428) a
329) b	330) c	331) c	332) b	429) c	430) b	431) a	432) d
333) a	334) b	335) a	336) a	433) b	434) b	435) a	436) b
337) c	338) b	339) a	340) b	437) b	438) b	439) d	440) c
341) a	342) c	343) d	344) d	441) a	442) a	443) d	444) c
345) d	346) b	347) d	348) a	445) c	446) a	447) c	448) b
349) a	350) c	351) d	352) d	449) b	450) c	451) d	452) d
353) d	354) b	355) b	356) b	453) c	454) b	455) d	456) c
357) a	358) c	359) d	360) a	457) d	458) b	459) a	460) a
361) b	362) c	363) c	364) b	461) b	462) b	463) b	464) c
365) d	366) b	367) b	368) a	465) d	466) d	467) d	468) d
369) b	370) a	371) d	372) a	469) b	470) d	471) b	472) c
373) c	374) c	375) d	376) c	473) c	474) c	475) b	476) d
377) b	378) b	379) c	380) a	477) c	478) c	479) d	480) c
381) a	382) a	383) d	384) c	481) b	482) a	483) a	484) c
385) d	386) d	387) d	388) a	485) d	486) a	487) c	488) a
389) a	390) b	391) a	392) c	489) c	490) d	491) b	492) a
393) d	394) c	395) a	396) d	493) c	494) d	495) c	496) d
397) a	398) c	399) c	400) c	497) b	498) b	499) c	500) b
401) c	402) a	403) c	404) a	501) b	502) c	503) a	504) d
405) d	406) c	407) a	408) d	505) c	506) b	507) c	508) c
409) d	410) d	411) b	412) b	509) d	510) c		
413) a	414) d	415) d	416) a				
417) c	418) c	419) b	420) c				

OSCILLATIONS

: HINTS AND SOLUTIONS :

1 **(c)**
The motion of sphere is simple harmonic. It's time period (T_0) is given by
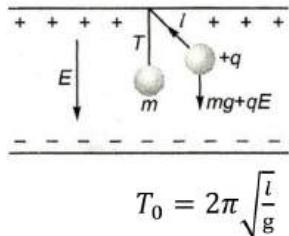
$$T_0 = 2\pi\sqrt{\frac{l}{g}} \qquad ...(i)$$

where l is length of string, g the acceleration due to gravity.
When sphere is placed in electric field. (E) force due to electric field acts on the sphere-1
$$F_E = qE = mg$$
where q is charge on sphere.
Hence, resultant acceleration is
$$g' = g + \frac{qE}{m}$$
$$\therefore \qquad T = 2\pi\sqrt{\frac{l}{g+\frac{qE}{m}}}$$
...(ii)

[Time period decreases]
Dividing Eq. (ii) by Eq. (i), we get
$$\frac{T}{T_0} = \sqrt{\frac{g}{g+\frac{qE}{m}}}$$

2 **(c)**
The time period of simple pendulum in air
$$T = t_0 = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$
...(i)
l, being the length of simple pendulum.
In water, effective weight of bob
w' = weight of bob in air – upthrust
$$\Rightarrow \qquad \rho V g_{eff} = mg - m'g$$
$$= \rho V g - \rho' V g = (\rho - \rho')Vg$$
where $\qquad \rho'$ = density of bob,
ρ = density of water

$$\therefore \qquad g_{eff} = \left(\frac{\rho-\rho'}{\rho}\right)g = \left(1-\frac{\rho'}{\rho}\right)g$$

$$\therefore \qquad t = 2\pi\sqrt{\left[\frac{l}{\left(1-\frac{\rho'}{\rho}\right)g}\right]}$$
...(ii)

Thus, $\qquad \dfrac{t}{t_0} = \sqrt{\left[\dfrac{1}{\left(1-\frac{\rho'}{\rho}\right)}\right]}$

$$= \sqrt{\left(\frac{1}{1-\frac{1000}{(4/3\times1000)}}\right)} = \sqrt{\left(\frac{4}{4-3}\right)} =$$

2
$$\Rightarrow \qquad t = 2t_0$$

3 **(b)**
PE varies from zero to maximum. It is always positive sinusoidal function

4 **(a)**
Let T_1, T_2 be the time period of shorter length and longer length pendulums respectively. Ads per question, $nT_1 = (n-1)T_2$;
So $n2\pi\sqrt{\dfrac{0.5}{g}} = (n-1)2\pi\sqrt{\dfrac{20}{g}}$
or $n = (n-1)\sqrt{40} \approx (n-1)6$
Hence, $5n = 6$
Hence, after 5 oscillations they will be in same phase

5 **(c)**
At centre $v_{max} \Rightarrow a\omega = a.\dfrac{2\pi}{T} = \dfrac{0.2\times2\pi}{0.01} = 40\pi$

6 **(d)**
$$F_1 = \frac{m4\pi^2 a}{\pi^2} \text{ and } F_2 = \frac{m4\pi^2 a}{T_2^2}$$
$$F = F_1 + F_2 = \frac{4\pi^2 ma}{T_1^2} + \frac{4\pi^2 ma}{T_2^2}$$
$$= 4\pi^2 ma\left(\frac{1}{T_1^2} + \frac{1}{T_2^2}\right)$$
Or $\dfrac{4\pi^2 ma}{T^2} = 4\pi^2 ma\left(\dfrac{1}{T_1^2} + \dfrac{1}{T_2^2}\right)$
Or $\dfrac{1}{T^2} = \dfrac{1}{T_1^2} + \dfrac{1}{T_2^2}$

Or $\frac{1}{T^2} = \frac{T_1^2 + T_2^2}{T_1^2 T_2^2}$ or $T^2 = \frac{T_1^2 T_2^2}{T_1^2 + T_2^2}$

7 (b)

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l \propto T^2$ [Equation of parabola]

8 (d)

Here, $m = 4kg; k = 800 Nm^{-1}; E = 4J$

In SHM, total energy is $E = \frac{1}{2}kA^2$

where A is the amplitude of oscillation

$\therefore 4 = \frac{1}{2} \times 800 \times A^2$

$A^2 = \frac{8}{800} = \frac{1}{100}$

$\Rightarrow A = \frac{1}{10}m = 0.1m$

Maximum acceleration, $a_{max} = \omega^2 A$

$= \frac{k}{m}A \qquad \left[\because \omega = \sqrt{\frac{k}{m}}\right]$

$= \frac{800 Nm^{-1}}{4kg} \times 0.1m = 20ms^{-2}$

9 (b)
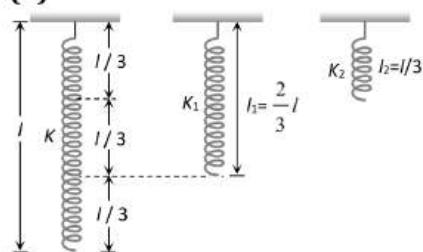
Force constant $(k) \propto \frac{1}{\text{Length of spring}}$

$\Rightarrow \frac{K}{K_1} = \frac{l_1}{l} = \frac{\frac{2}{3}l}{l} \Rightarrow K_1 = \frac{3}{2}K$

10 (b)

Total energy $U = \frac{1}{2}Ka^2$

11 (b)

$\omega = \sqrt{k/m} = \sqrt{\frac{4.84}{0.98}} = 2.22 \, rad/s$

12 (a)

For resonance amplitude must be maximum which is possible only when the denominator of expansion is zero

i.e. $a\omega^2 - b\omega + c = 0 \Rightarrow \omega = \frac{+b \pm \sqrt{b^2 - 4ac}}{2a}$

For a single resonant frequency, $b^2 = 4ac$

13 (a)

Inside the mine g decreases

Hence from $T = 2\pi\sqrt{\frac{l}{g}}$; T increase

14 (c)

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow \frac{l}{T^2} = \frac{g}{4\pi^2} = \text{constant}$

15 (a)

KE of a body undergoing SHM is given by

$KE = \frac{1}{2}m\omega^2 A^2 \cos^2\omega t$ and $KE_{max} = \frac{m\omega^2 A^2}{2}$

[symbols represent standard quantities]

From given information

$$KE = (KE_{max}) \times \frac{75}{100}$$

$\Rightarrow \qquad \frac{m\omega^2 A^2}{2}\cos^2\omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4}$

$\Rightarrow \qquad \cos\omega t = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \qquad \omega t = \frac{\pi}{6}$

$\Rightarrow \qquad \frac{2\pi}{T} \times t = \frac{\pi}{6}$

$\Rightarrow \qquad t = \frac{T}{12} = \frac{1}{6}s$

16 (d)

When spring is cut into two equal parts then spring constant of each part will be $2K$ and so using $n \propto \sqrt{K}$, new frequency will be $\sqrt{2}$ times, i.e. $f_2 = \sqrt{2}f_1$

17 (a)

Time period of pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \qquad\qquad\qquad T \propto \sqrt{l}$

18 (d)

Let x be the point where K.E. = P.E.

Hence $\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}m\omega^2 x^2$

$\Rightarrow 2x^2 = a^2 \Rightarrow \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \, cm$

19 (d)

The periodic time of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When taken to height $2R$.

$$g' = g\left(1 + \frac{h}{R_e}\right)^2$$

$$= g\left(1 + \frac{2R}{R}\right)^{-2} = g(3)^{-2}$$

$\therefore \qquad \frac{T_1}{T_2} = \sqrt{\frac{1}{3^2}}$

$\Rightarrow \qquad T_2 = 3T_1 \Rightarrow \frac{T_1}{T_2} = \frac{1}{3}$

20 (d)
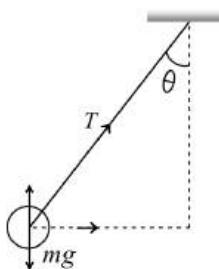
$T \sin \theta = mL \sin \theta \omega^2$

$324 = 0.5 \times 0.5 \times \omega^2$

$\Rightarrow \omega^2 = \dfrac{324}{0.5 \times 0.5}$

$\Rightarrow \omega = \sqrt{\dfrac{324}{0.5 \times 0.5}}$

$\Rightarrow \omega = \dfrac{18}{0.5} = 36 \, rad/sec$

21 **(b)**

As here two masses are connected by two springs, this problem is equivalent to the oscillation of a reduced mass m_r of a spring of effective spring constant

$T = 2\pi \sqrt{\dfrac{m_r}{K_{eff.}}}$

Here $m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{m}{2} \Rightarrow K_{eff.} = K_1 + K_2 = 2K$

$\therefore n = \dfrac{1}{2\pi}\sqrt{\dfrac{K_{eff.}}{m_r}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2K}{m} \times 2} = \dfrac{1}{\pi}\sqrt{\dfrac{K}{m}} = \dfrac{1}{\pi}\sqrt{\dfrac{0.1}{0.1}}$

$= \dfrac{1}{\pi} \, Hz$

22 **(b)**

The acceleration of a particle in SHM is,

$\alpha_{\max} = -\omega^2 A$

Where ω is angular velocity and A the amplitude.

Given, $y = 2 \sin\left[\dfrac{\pi t}{2} + \emptyset\right]$

...(i)

Standard equation of a wave in SHM is

$y = A \sin(\omega t + \emptyset)$

...(iii)

Comparing Eq. (i) with Eq. (ii), we get

$A = 2 \, cm, \omega = \dfrac{\pi}{2}$

$\therefore \qquad \alpha_{\max} = -\left(\dfrac{\pi}{2}\right)^2 \times 2$

$= \dfrac{\pi^2}{2} \, cms^{-2}$

23 **(c)**

When $t = 0, x = r \cos \dfrac{\pi \times 0}{2} = r$;

When $t = 3s, x = r \cos \dfrac{\pi \times 3}{2} = 0$

Here $\omega = \dfrac{\pi}{2}$ or $\dfrac{2\pi}{T} = \dfrac{\pi}{2}$ or $T = 4s$

$\therefore$ In 3 sec, the particle goes from one extreme to other extreme and then back to mean position. So the distance travelled $= 2r + r = 3r$

24 **(a)**

Time period $T = 2\pi \sqrt{\dfrac{L}{g}}$

25 **(b)**

Force constant of a spring is given by $F = kx$

$6.4 = k(0.1)$ or $k = 64N/m$

$\because T = 2\pi \sqrt{\dfrac{m}{k}} \Rightarrow \dfrac{\pi}{4} = 2\pi \sqrt{\dfrac{m}{64}}; \dfrac{m}{64} = \left(\dfrac{1}{8}\right)^2; m = 1kg$

26 **(d)**

$\tau_A = \tau_B = \left(mg\dfrac{L}{2}\sin \theta + M \, gL \sin \theta\right)$

$=$ Restoring torque about point O.

In case A, moment of inertia will be more. Hence, angular acceleration ($\alpha = \tau/I$) will be less. Therefore angular frequency will be less. Note Question is difficult because this type of SHM is rarely.
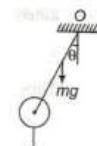
27 **(d)**

Function of wrist watch depends upon spring action so it is not effected by gravity but pendulum clock has time period, $T = 2\pi \sqrt{\dfrac{l}{g}}$.

During free fall effective acceleration becomes zero, so time period comes out to be infinity, i. e., the clock stops

28 **(d)**

If m is the mass, r is the amplitude of oscillation, then maximum kinetic energy,

$K_0 = \dfrac{1}{2}m\omega^2 r^2$ or $r = \left(\dfrac{2K_0}{m\omega^2}\right)^{\frac{1}{2}}$

The displacement equation can be

$y = r \sin \omega t = \left(\dfrac{2K_0}{m\omega}\right)^{\frac{1}{2}} \sin \omega t$

29 **(c)**
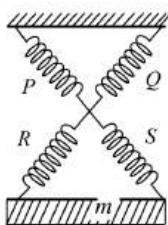
Spring P and Q, R and S are in parallel
then, $x = k + k = 2k$ [for P, Q]
and $y = k + k = 2k$ [for R, S]
x and y both in series

$\therefore \dfrac{1}{k''} = \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{k}$

Time period $T = 2\pi\sqrt{\dfrac{m}{k''}} = 2\pi\sqrt{\dfrac{m}{k}}$

30 (a)

$T = 2\pi\sqrt{\dfrac{m}{k}}$

$= 2\pi\sqrt{\dfrac{0.2}{80}} = 0.315$

31 (b)

Total potential energy $= 0.04\,J$
Resting potential energy $= 0.01\,J$
Maximum kinetic energy $= (0.04 - 0.01)$

$= 0.03J = \dfrac{1}{2}m\,\omega^2 a^2 = \dfrac{1}{2}ka^2$

$0.03 = \dfrac{1}{2} \times k \times \left(\dfrac{20}{1000}\right)^2$

$k = 0.06 \times 2500\,N/m = 150\,N/m$

32 (a)
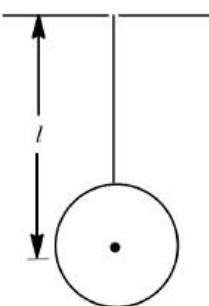
Spherical hollow ball
filled with water

$T = 2\pi\sqrt{\dfrac{l}{g}}$
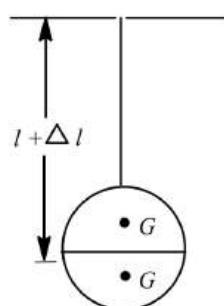
Spherical hollow ball
half filled with water

$T_1 = 2\pi\sqrt{\dfrac{l+\Delta l}{g}}$
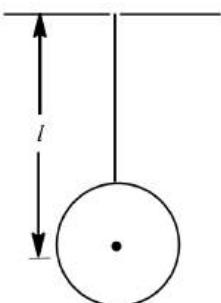
Spherical hollow ball

$T_2 = 2\pi\sqrt{\dfrac{l}{g}}$

and $\qquad\qquad T_1 > T_2$

Hence, time period first increases and then decreases to the original value.

33 (d)

$h = l\,(1 - \cos\theta)$
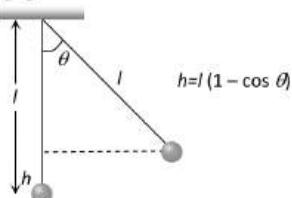
Kinetic energy will be maximum at mean position. From law of conservation of energy maximum kinetic energy at mean position = Potential energy at displaced position

$\Rightarrow K_{max} = mgh = mgl(1 - \cos\theta)$

34 (a)

In this case time period of pendulum becomes
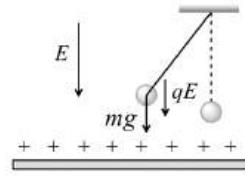
$T'' = 2\pi\sqrt{\dfrac{l}{\left(g + \dfrac{qE}{m}\right)}}$

$\Rightarrow T'' < T$

35 (c)

$y = 0.25 \sin 200t;$

Speed, $\dfrac{dy}{dt} = 0.25 \times 200 \cos 200t$

Max. speed $= 0.25 \times 200 = 50 \, \text{cm s}^{-1}$

36 (d)

In simple harmonic motion

$y = a \sin \omega t$ and $v = a\omega \cos \omega t$ from this have

$\dfrac{y^2}{a^2} + \dfrac{v^2}{a^2\omega^2} = 1$, which is a equation of ellipse

37 (b)

$x = 3 \sin \omega t + 4 \sin(\omega t + \pi/3)$

Comparing it with the equations

$x = r_1 \sin \omega t + r_2 \sin(\omega t + \phi)$

We have, $r_1 = 3$ cm, $r_2 = 4$cm and $\phi = \pi/3$

The amplitude of combination is

$r = \sqrt{r_1^2 + r_2^2 + 2r_1r_2 \cos \phi}$

$= \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos \pi/3}$

$= \sqrt{37} = 6$ cm

38 (c)

Time period is independent of mass of pendulum

39 (b)

A total restoring force, $F = kX = mg$

Or $k = mg/X$

Total mass that oscillates $= (M + m)$

$\therefore \; T = 2\pi \sqrt{\dfrac{(M + m)}{mg/X}} = 2\pi \sqrt{\dfrac{(M + m)X}{mg}}$

40 (b)

Let x be the maximum extension of the spring. From energy conservation
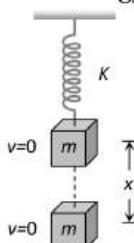
Loss in gravitational potential energy
= Gain in potential energy of spring

$Mgx = \dfrac{1}{2}Kx^2$

$\Rightarrow x = \dfrac{2Mg}{K}$

41 (c)

Under the influence of one force $F_1 = m\omega_1^2 y$ and under the action of another force, $F_2 = m\omega_2^2 y$

Under the action of both the forces $F = F_1 + F_2$

$\Rightarrow m\omega^2 y = m\omega_1^2 y + m\omega_2^2 y$

$\Rightarrow \omega^2 = \omega_1^2 + \omega_2^2 \Rightarrow \left(\dfrac{2\pi}{T}\right)^2 = \left(\dfrac{2\pi}{T_1}\right)^2 + \left(\dfrac{2\pi}{T_2}\right)^2$

$\Rightarrow T = \sqrt{\dfrac{T_1^2 T_2^2}{T_1^2 + T_2^2}} = \sqrt{\dfrac{\left(\frac{4}{5}\right)^2 \left(\frac{3}{5}\right)^2}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}} = 0.48s$

42 (b)

$T = 2\pi \sqrt{\dfrac{m}{k}}, T' = 2\pi \sqrt{\dfrac{m}{4k}} = \dfrac{T}{2} = \dfrac{5}{2}s = 2.5$ s

43 (c)

If v and v' are the velocities of the block of mass M and $(M + m)$ while passing from the mean position when executing SHM

Using law of conservation of linear momentum, we have

$mv = (M + m)v'$ or $v' = mv/(M + m)$

Also, maximum PE= maximum KE

$\therefore \dfrac{1}{2} k A'^2 = \dfrac{1}{2}(M + m)v'^2$

or $A' = \left(\dfrac{M+m}{k}\right)^{1/2} \times \dfrac{mv}{(M+m)}$

$= \dfrac{mv}{\sqrt{(M + m)k}}$

44 (a)
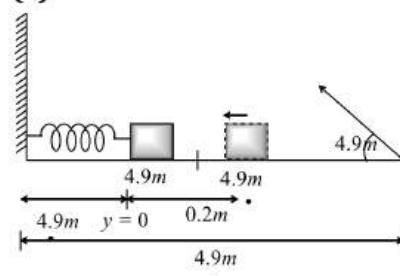
The block is released from A

$x = 4.9m + (0.2m) \sin\left(\omega t + \dfrac{\pi}{2}\right)$

at $t = 1s; x = 5m$

so range of projectile will be $5m$

Now $5 = \dfrac{v^2 \sin 90°}{g} \Rightarrow v^2 = 50 \Rightarrow v = \sqrt{50}$

45 (c)

On comparing with standard equation $\dfrac{d^2y}{dt^2} + \omega^2 y = 0$

we get $\omega^2 = K \Rightarrow \omega = \dfrac{2\pi}{T} = \sqrt{K} \Rightarrow T = \dfrac{2\pi}{\sqrt{K}}$

46 (c)

$y = A \sin\left(\dfrac{2\pi}{T}\right)t$

$\dfrac{A}{2} = A \sin\left(\dfrac{2\pi}{T}\right)t = \dfrac{2\pi}{T}t = \pi/6$

Time period $t = \dfrac{T}{12} = \dfrac{6}{12} = \dfrac{1}{2}$ s

47 (d)

Potential energy of particle performing SHM is given by: $PE = \dfrac{1}{2}m\omega^2 y^2$, i.e., it varies

parabolically such that at mean position it becomes zero and maximum at extreme positions

49 **(c)**
A particle oscillating under a force $\vec{F} = -k\vec{x} = b\vec{v}$ is a damped oscillator. The first term $-k\vec{x}$ represents the restoring force and second term $-b\vec{v}$ represents the damping force

50 **(c)**
Effective force constant is equal to the reciprocal of the sum of individual force constant, hence
$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots$$
Given, $k_1 = k, k_2 = 2k, k_3 = 3k, \ldots..$
$\therefore \quad \frac{1}{k_e} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \cdots$

The given series a geometric progression series, hence sum is
$$S_\infty = \frac{a}{1-r}$$
where a is first term of series and r the common difference.
$$\Rightarrow \quad \frac{1}{k_e} = \frac{1}{k} \times \frac{1}{\left(1 - \frac{1}{2}\right)} = \frac{2}{k} \Rightarrow k_e = \frac{k}{2}$$

51 **(a)**

Time period $\qquad T = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}}$
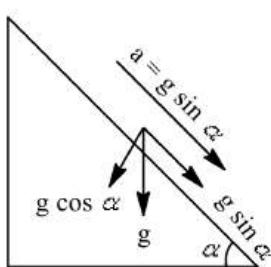
$$T = 2\pi\sqrt{\frac{l}{g\cos\alpha}}$$

52 **(d)**
$$x_1 + x_2 = A \text{ and } k_1 x_1 = k_2 x_2 \quad \text{or} \quad \frac{x_1}{x_2} = \frac{k_2}{k_1}$$
Solving these equations, we get
$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right) A$$

53 **(c)**
The effective acceleration in a lift descending with acceleration $\frac{g}{3}$ is $g_{eff} = g - \frac{g}{3} = \frac{2g}{3}$
$$\therefore T = 2\pi\sqrt{\left(\frac{L}{g_{eff}}\right)} = 2\pi\sqrt{\left(\frac{L}{2g/3}\right)} = 2\pi\sqrt{\left(\frac{3L}{2g}\right)}$$

54 **(b)**

Time period of simple pendulum is given by
$$T = 2\pi\sqrt{\frac{l}{g}}$$
...(i)
When the lift is moving up with an acceleration a, then time period becomes
$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$
Here, $\quad T' = \frac{T}{2}$
$$\Rightarrow \quad \frac{T}{2} = 2\pi\sqrt{\frac{l}{g+a}}$$
...(ii)
Dividing Eq.(ii) by Eq. (i), we get
$$a = 3g$$

56 **(d)**
$$n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{K_1 K_2}{(K_1 + K_2)m}}$$

57 **(b)**
$m_1 = 1 \text{ kg}$, extension $l_1 = 5 \text{ cm} = 5 \times 10^2 \text{ m}$
$\therefore \qquad m_1 g = k l_1$
$k =$ force constant of the spring
$$k = \frac{m_1 g}{l_1} = \frac{1 \times 10}{5 \times 10^{-2}} = 200 \text{ Nm}^{-1}$$
Time period of the block of mass 2 kg.
$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 2\pi \times \frac{1}{10} = \frac{\pi}{5} \text{ s}$$
Maximum velocity $v_{\max} = A\omega$
where $A =$ Amplitude
$$= 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$
$$v_{\max} = A \times \frac{2\pi}{T} = 10 \times 10^{-2} \times \frac{2\pi}{\pi/5}$$
$$= 10^{-1} \times 2 \times 5 = 1 \text{ ms}^{-1}$$

58 **(c)**
Torque about $P = -(kx)\frac{L}{2} + \left(-kx\frac{L}{2}\right) = -kxL = -k\frac{L^2}{2}\theta$
For small angle θ, $x = \frac{L}{2}\theta$; $\tau = -I\alpha$
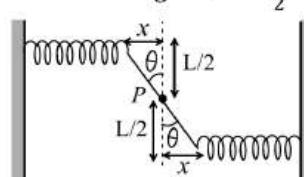
$$\Rightarrow -\frac{KL^2}{2}\theta = \frac{ML^2}{12}\alpha$$
$$\Rightarrow \frac{-6K\theta}{M} = \alpha$$
$$\Rightarrow \omega = \sqrt{\frac{6K}{M}} \text{ and } f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

59 (c)

If first equation is

$$y_1 = a_1 \sin \omega t \Rightarrow \sin \omega t = \frac{y_1}{a_1}$$

...(i)

Then second equation will be

$$y_2 = a_2 \sin(\omega t + \frac{\pi}{2})$$

$$= a_2[\sin \omega t \cos \frac{\pi}{2} + \cos \omega t \sin \frac{\pi}{2}] = a_2 \cos \omega t$$

$$\Rightarrow \quad \cos \omega t = \frac{y_2}{a_2}$$

...(ii)

By squaring and adding Eqs. (i) and (ii)

$$\sin^2 \omega t + \cos^2 \omega t = \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2}$$

$$\Rightarrow \quad \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} = 1;$$

This is equation of an ellipse.

60 (c)

By using conservation of mechanical energy

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow x = v\sqrt{m/k}$$

61 (a)

When block (C) strikes the block (A), then it begins to oscillate, whose time period

$$T = 2\pi \sqrt{\frac{m}{2k}}$$
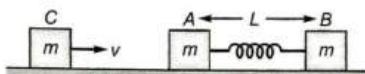
Compression $x = vT = v \times 2\pi \sqrt{\frac{m}{2k}}$

$$\therefore \qquad x \propto v\sqrt{\frac{m}{2k}}$$

62 (d)

Spring is cut into two equal halves so spring constant of each part $= 2k$

These parts are in parallel so $K_{eq} = 2K + 2K = 4K$

Extension force (i.e. W) is same hence by using $F = kx$

$$\Rightarrow 4k \times x' = kx \Rightarrow x' = \frac{x}{4}$$

63 (a)

On the inclined plane, the effective acceleration due to gravity

$$g' = g\cos 30°$$
$$= g \times \sqrt{3}/2$$

$$\therefore \quad T = 2\pi \sqrt{\frac{1}{g'}} = 2\pi \sqrt{\frac{2l}{\sqrt{3}g}}$$

64 (d)

Standard equation of S.H.M. $\frac{d^2y}{dt^2} = -\omega^2 y$, is not satisfied by $y = a \tan \omega t$

65 (c)

$$v = \omega\sqrt{(a^2 - y^2)} = 2\sqrt{60^2 - 20^2} = 113mm/s$$

66 (d)

Let the distance of vertical disc c of block be pushed in liquid, when block is floating, then Buoyancy force

$= abxx_\omega, g = abxg$

The mass of piece of wood$= abcd$

So acceleration$= -ab\,xg/abcd = -\left(\frac{g}{cd}\right)x$

Hence, time period, $T = 2\pi \sqrt{\frac{dc}{g}}$

67 (d)

When the bob is immersed in water its effective weight

$$= \left(mg - \frac{m}{\rho}g\right) = mg\left(\frac{\rho - 1}{\rho}\right)$$

$$\therefore g_{eff} = g\left(\frac{\rho - 1}{\rho}\right)$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g_{eff}}} \Rightarrow T' = T\sqrt{\frac{\rho}{(\rho - 1)}}$$

68 (d)

$$y = A \sin \omega t = \frac{A\sin 2\pi}{T}t \Rightarrow \frac{A}{2}$$

$$= A\sin\frac{2\pi t}{T} \Rightarrow t = \frac{T}{12}$$

69 (d)

Time period of harmonic oscillator is independent of the amplitude of oscillation. Energy of oscillation is

$$E = \frac{1}{2}m\omega^2 a^2 \; ie, E \propto a^2$$

So if a is double, E becomes four times.

70 (a)

On a planet, if a body dropped initial velocity ($u = 0$) from a height h and takes time t to reach the ground then $h = \frac{1}{2}g_P t^2 \Rightarrow g_P = \frac{2h}{t^2} = \frac{2 \times 8}{4} = 4\,m/s^2$

Using $T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T = 2\pi \sqrt{\frac{1}{4}} = \pi = 3.14\,sec$

71 (a)

$$y = kt^2$$
$$\frac{d^2y}{dt^2} = 2k$$

or $a_y = 2 \text{ ms}^{-2}$

(as $k = 1 \text{ ms}^{-2}$)

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

and $T_2 = 2\pi\sqrt{\frac{l}{g+a_y}}$

∴ $\frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{6}{5}$

72 (b)

$v_x = A(1 - \cos px)$

$F = -\frac{dv}{dx} = -Ap\sin px$

For small (x)

$F = -Ap^2 x$

$a = -\frac{Ap^2}{m}x$

$a = \omega^2 x$

$\omega = \sqrt{\frac{Ap^2}{m}}$

∴ $T = 2\pi\sqrt{\frac{m}{Ap^2}}$

73 (c)

$v_{\max} = a\omega = a\frac{2\pi}{T}$

$\Rightarrow a = \frac{v_{\max}T}{2\pi} = \frac{15 \times 628 \times 10^{-3}}{2 \times 3.14} = 1.5 \, cm$

74 (b)

The time period of a pendulum of length l, is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$\Rightarrow \qquad l = g\frac{T^2}{4\pi^2}$

Since, $T = 2 \, s$ (for second's pendulum)

∴ $l_1 = \frac{g_1(2)^2}{4\pi^2} = \frac{g_1}{\pi^2}$; $l_2 = \frac{g_2(2)^2}{4\pi^2} = \frac{g_2}{\pi^2}$

Since, length is decreased, g_2 is less than g_1

∴ $l_1 - l_2 = \frac{g_1 - g_2}{\pi^2}$

$\Rightarrow (l_1 - l_2)\pi^2 = g_1 - g_2$

$0.3 \times 10 = g_1 - g_2$

∴ $g_2 = 981 - 3 = 978 \text{ cms}^{-2}$

75 (d)

In S.H.M. at mean position velocity is maximum

So $v = a\omega$ (maximum)

76 (a)

As the girl stands up, the effective length of pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T will decrease.

77 (a)

$$T = 2\pi\sqrt{\frac{L}{g}}$$
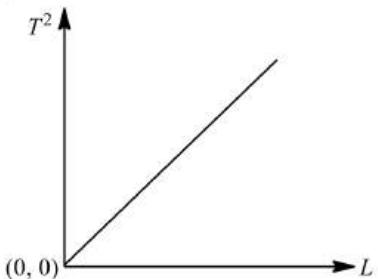
or $T \propto \sqrt{L}$ or $T^2 \propto L$

It is linear relation between T^2 and l hence the graph between T^2 and L is a straight line passing through the origin.

78 (c)

The motion of a planet around the sun is a periodic motion but not a simple harmonic motion. All other given motions are the examples of simple harmonic motion

80 (a)

Fig. (i) alone represents damped SHM

81 (c)

When the bob falls through a vertical height of 1m, the velocity acquired at the lowest point,

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1} = \sqrt{20} \text{ ms}^{-1}$

Centrifugal force$= \frac{mv^2}{r} = \frac{0.01 \times 20}{1} = 0.20$ N

Net tension=weight+centrifugal force

$= (0.01 \times 10 + 0.20) = 0.30$ N

82 (c)

Mass (m)=20 g=0.02 kg

Frequency $(f)=\frac{5}{\pi}$ Hz

Time period of a loaded spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency $(f) = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\frac{5}{\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{0.02}}$

or $10 = \sqrt{\frac{k}{0.02}}$

or $100 = \frac{k}{0.02}$

∴ $k = 2 \text{ Nm}^{-1}$

83 (c)

$y_1 = a \sin(\omega t - kx);$

$y_2 = b \cos\left(\omega t - \dfrac{k}{x}\right) = b \sin\left(\omega t - \dfrac{k}{x} + \pi/2\right)$

∴ Phase difference $= \left(\omega t - \dfrac{k}{x} + \pi/2\right) - (\omega t - kx)$

$= \pi/2$

84 (c)

When a mass m is placed on mass M, the new system is of mass $= (M + m)$, attached to the spring. New time period of oscillation,

$T' = 2\pi \sqrt{\dfrac{M + m}{k}}$

$T = 2\pi \sqrt{\dfrac{M}{k}}$

Let v =velocity of the mass M while passing through the mean position.

v' =Velocity of the mass $(M + m)$, while passing through the mean position.

According to law of conservation of linear momentum $Mv = (M + m)v'$

At mean position, $v = A\omega$ and $v' = A'\omega'$

∴ $\quad MA\omega = (m + m)A'\omega$

or $\ A' = \left(\dfrac{M}{M+m}\right)\dfrac{\omega}{\omega}, A = \dfrac{M}{M+m} \times \dfrac{T'}{T} \times A$

$= \left(\dfrac{M}{M+m}\right) \times \sqrt{\dfrac{M+m}{M}} \times A$

$= A\sqrt{\dfrac{M}{M+m}}$

85 (c)

$T = 2\pi \sqrt{\dfrac{M}{k}}$ when mass is increased by m then

...(i)

$$T = 2\pi \sqrt{\dfrac{M+m}{k}}$$

$\Rightarrow \qquad \dfrac{5T}{3} = 2\pi \sqrt{\dfrac{M+m}{k}}$

...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\dfrac{3}{5} = \sqrt{\dfrac{M}{M+m}}$$

$$\dfrac{9}{25} = \dfrac{M}{M+m}$$

$\Rightarrow \qquad 9M + 9m = 25M$

$\Rightarrow \qquad 16M = 9m$

$$\dfrac{m}{M} = \dfrac{16}{9}$$

86 (a)

87 (a)

$T \propto \dfrac{1}{\sqrt{k}} \Rightarrow T_1 : T_2 : T_3 = \dfrac{1}{\sqrt{k}} : \dfrac{1}{\sqrt{k/2}} : \dfrac{1}{\sqrt{2k}} = 1 : \sqrt{2} : \dfrac{1}{\sqrt{2}}$

$y_1 = 4 \sin\left(4\pi t + \dfrac{\pi}{2}\right) = 4 \cos 4\pi t$

$y_2 = 3 \cos(4\pi t) = 3 \cos 4\pi t$

The phase difference = 0, both are along the same line

∴ $A^2 = 4^2 + 3^2 + 2 \times 4 \times 3 \cos 0°$

$A^2 = (4 + 3)^2 \Rightarrow A = 7$

The resultant amplitude is 7 units

88 (b)

$n = \dfrac{1}{2\pi} \sqrt{\dfrac{g}{l}} \Rightarrow n \propto \dfrac{1}{\sqrt{l}} \Rightarrow \dfrac{n_1}{n_2} = \sqrt{\dfrac{l_2}{l_1}} = \sqrt{\dfrac{L_2}{2L_2}}$

$\Rightarrow \dfrac{n_1}{n_2} = \dfrac{1}{\sqrt{2}} \Rightarrow n_2 = \sqrt{2}n_1 \Rightarrow n_2 > n_1$

Energy $E = \dfrac{1}{2} m\omega^2 a^2 = 2\pi^2 m n^2 a^2$

$\Rightarrow \dfrac{a_1^2}{a_2^2} = \dfrac{m_2 n_2^2}{m_1 n_1^2} \qquad [\because E \text{ is same}]$

Given $n_2 > n_1$ and $m_1 = m_2 \Rightarrow a_1 > a_2$

89 (b)

The two spring on left side having spring constant of $2k$ each are in series, equivalent constant is $\dfrac{1}{\left(\frac{1}{2k} + \frac{1}{2k}\right)} = k$. The two springs on right hand side of mass M are in parallel. Their effective spring constant is $(k + 2k) = 3k$

Equivalent spring constants of value k and $3k$ are in parallel and their net value of spring constant of all the four springs is $k + 3k = 4k$

∴ Frequency of mass is $n = \dfrac{1}{2\pi} \sqrt{\dfrac{4k}{M}}$

90 (a)

For S.H.M. $F = -kx$

∴ Force = Mass × Acceleration $\propto -x$

$\Rightarrow F = -Akx$; where A and k are positive constants

91 (c)

When lift accelerates upwards, then effective acceleration on the pendulum

$$g_{eff} = g + \dfrac{g}{3} = \dfrac{4g}{3}$$

∴ Time period $T' = 2\pi \sqrt{\dfrac{l}{g_{eff}}} = 2\pi \sqrt{\dfrac{l}{4g/3}}$

$= \dfrac{\sqrt{3}}{2} \cdot 2\pi \sqrt{\dfrac{l}{g}}$

$= \dfrac{\sqrt{3}}{2} T$

92 (a)

When particle is at $x = 2$, the displacement is $y = 4 \sim 2 = 2$ cm. If r is the time taken by the particle to go from $x = 4$ cm to $x = 2$ cm, then

$$y = a \cos \omega t = a \cos \frac{2\pi t}{T} = a \cos \frac{2\pi t}{1.2}$$

or $\cos \frac{2\pi t}{1.2} = \frac{y}{a} = \frac{2}{4} = \frac{1}{2} = \cos \frac{\pi}{3}$

or $\frac{2t}{1.2} = \frac{1}{3}$ or $t = \frac{1.2}{6} = 0.26$

time taken to move from $x = +2$ cm to $x = +4$ cm and back again

$= 2t = 2 \times 0.2 \, \text{s} = 0.4 \, \text{s}$

93 (a)

Under forced oscillations, the body will vibrate with the frequency of the driving force

94 (c)

$E = \frac{1}{2} m \omega^2 a^2 \Rightarrow \frac{E'}{E} = \frac{a'^2}{a^2} \Rightarrow \frac{E'}{E}$

$$= \frac{\left(\frac{3}{4} a\right)^2}{a^2} \quad \left(\because a' = \frac{3}{4} a\right)$$

$\Rightarrow E' = \frac{9}{16} E$

95 (b)

Let at any instant, cube is at a depth x from the equilibrium position then net force acting on the cube = upthrust on the portion of length x

$$F = -\rho l^2 x g = -\rho l^2 g x$$

...(i)

Negative sign shows that, force is opposite to x. Hence equation of SHM

$$F = -kx$$

...(ii)

Comparing Eqs. (i) and (ii)
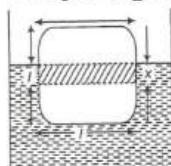
$k = \rho l^2 g$

$T = 2\pi \sqrt{\frac{m}{k}}$

$= 2\pi \sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi \sqrt{\frac{ld}{\rho g}}$

96 (a)

We know that during SHM, the restoring force is proportional to the displacement from equilibrium position. Hence restoring force is maximum when the displacement is maximum at its extreme position

97 (b)

Kinetic energy in SHM

$\text{KE} = \frac{1}{2} m \omega^2 (A^2 - x^2)$

At $x = \frac{A}{2}$

$\text{KE} = \frac{1}{2} m \omega^2 \left(A^2 - \frac{A^2}{4}\right) = \frac{3}{4}\left(\frac{1}{2} m \omega^2 A^2\right)$

$= \frac{3}{4} \times$ Total energy of the particle

98 (c)

Displacement equation

$$y = A \sin \omega t - B \cos \omega t$$

Let $A = a \cos \theta$ and $B = a \sin \theta$

So, $A^2 + B^2 = a^2$

$\Rightarrow a = \sqrt{A^2 + B^2}$

Then, $y = a \cos \theta \sin \omega t - a \sin \theta \cos \omega t$

$y = a \sin(\omega t - \theta)$

which is the equation of simple harmonic oscillator

The amplitude of the oscillator

$$= a = \sqrt{A^2 + B^2}$$

99 (d)

$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{\frac{l}{g}}$, it is does not depend upon mass

100 (c)

When $t = 1 \, s, y_1 = r \sin \omega \times 1 = r \sin \omega$

When $t = 2 \, s, y_2 = r \sin \omega \times 2 = r \sin 2\omega$

$\therefore \frac{y_1}{y_2} = \frac{r \sin \omega}{r \sin 2\omega}$

$= \frac{1}{2 \cos \omega} = \frac{1}{2 \cos 2\pi/T}$

$= \frac{1}{2 \cos 2\pi/8}$

$= \frac{1}{2 \cos \pi/4}$

$= \frac{1}{2(1/\sqrt{2})} = \frac{1}{\sqrt{2}}$

$\therefore y_2 = \sqrt{2} y_1$

Distance converted in 2nd second

$= y_2 - y_1 = (\sqrt{2} - 1) y_1$

$\therefore$ Ratio $= 1 : (\sqrt{2} - 1)$

102 (b)

$$T = 2\pi \sqrt{\frac{l}{g}}$$

and $2T = 2\pi \sqrt{\frac{l}{g}}$

Then $\frac{1}{2} = \sqrt{\frac{l}{l'}}$ or $l' = 4l$

103 **(b)**

Time period of spring

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

k, being the force constant of spring
For first spring

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)}$$

...(i)
For second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)}$$

...(ii)
The effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

∴ Time period of combination

$$T = 2\pi\sqrt{\left(\frac{m(k_1+k_2)}{k_1 k_2}\right)}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1 k_2}$$

...(iii)
Form Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

$$t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow \quad t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1+k_2)}{k_1 k_2}$$

$$\therefore \quad t_1^2 + t_2^2 = T^2$$

[from Eq. (iii)]

104 **(b)**

Let h be the depth of bottle in water then

$$A h \rho g = mg \quad \text{or} \quad h = \frac{m}{A\rho} = \frac{200}{50 \times 1} = 4 \, cm$$

$$T = 2\pi\sqrt{\frac{h}{g}} \quad \text{or} \quad v = \frac{1}{2} = \frac{1}{2\pi}\sqrt{\frac{g}{h}}$$

$$= \frac{7}{2 \times 22} = \sqrt{\frac{980}{4}} = 2.5 \, \text{Hz}$$

105 **(d)**

$$T = 8s, \omega = \frac{2\pi}{T} = \left(\frac{\pi}{4}\right) \text{rads}^{-1}$$

$$x = A \sin \omega t$$

$$\therefore \quad a = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$$

Substituting $t = \frac{4}{3}$ s, we get

$$a = -\left(\frac{\sqrt{3}}{32}\pi^2\right) \text{cms}^{-2}$$

106 **(a)**

$$x = (2 \times 10^{-2}) \cos \pi t$$

Here, $a = 2 \times 10^{-2} \text{m} = 2 \, cm$
At $t = 0, x = 2 \, cm$, *ie*, the object is at positive extreme, so to acquire maximum speed (*ie*, to reach mean position) it takes $\frac{1}{4}$ th of time period.

∴ Required time $= \frac{T}{4}$

where $\omega = \frac{2\pi}{T} = \pi$

$\Rightarrow \quad T = 2 \, s$

So, required time $= \frac{T}{4} = \frac{2}{4} = 0.5 \, s$

107 **(b)**

For getting horizontal range, there must be some inclination of spring with ground to project ball
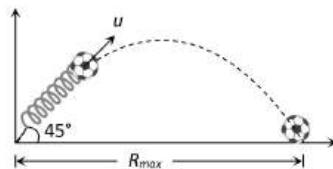
$\Rightarrow R_{\max} = \frac{u^2}{g}$ ...(i)

But K.E. acquired by ball = P.E. of spring gun

$\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}kx^2 \Rightarrow u^2 = \frac{kx^2}{m}$ ... (ii)

From equation (i) and (ii)

$$R_{\max} = \frac{kx^2}{mg} = \frac{600 \times (5 \times 10^{-2})^2}{15 \times 10^{-3} \times 10} = 10m$$

108 **(d)**

Using acceleration $A = -\omega^2 x$
$At - x_{\max} A$ will be maximum and positive

109 **(a)**

$$f = \frac{1}{T} = \frac{1}{0.04} = 25 \, Hz$$

110 **(d)**

$$g' = \sqrt{g^2 + a^2}$$
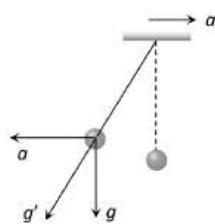
111 (b)

When the ball of mass m falls from a height h, it reaches the surface of earth in time $t = \sqrt{2h/g}$. Its velocity is $v = \sqrt{2gh}$. It then moves in to the tunnel and reaches on the other side of earth and goes again upto a height h from that side of earth. The ball again returns back and thus executes periodic motion. Outside the earth ball crosses distance h four times.

When the ball is in the tunnel at distance x from the centre of the earth, then gravitational force acting on ball is

$$F = \frac{Gm}{x^2} \times \left(\frac{4}{3}\pi x^2 \rho\right) = G \times \left(\frac{4}{3}\pi\rho\right)mx$$

Mass of the earth, $M = \frac{4}{3}\pi R^2 \rho$

or $\frac{4}{3}\pi\rho = \frac{M}{R^3}$

$\therefore \quad F = \frac{GMmx}{R^3} \quad ie, F \propto x$

As this F is directed towards the centre of earth ie, the mean position so the ball will execute periodic motion about the centre of earth
Here inertia factor=mass of ball= m

Spring factor= $\frac{GMm}{R^3} = \frac{gm}{R}$

$\therefore$ time period of oscillation of ball in the tunnel is

$$T' = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{gm/R}} = 2\pi\sqrt{\frac{R}{g}}$$

Time spent by ball outside the tunnel on both the sides will be $= 4\sqrt{2h/g}$

Therefore, total time period of oscillation of ball is

$$= 2\pi\sqrt{\frac{R}{g}} + 4\sqrt{\frac{2h}{g}}$$

112 (b)

When a little mercury is drained off, the position of $c.g.$ of ball falls ($w.r.t.$ fixed end) so that effective length of pendulum increases hence T increases

113 (d)

Here, the state of maximum amplitude of the oscillation is a measure of resonance.

114 (c)

Loss of potential energy in coming from A to B

$$= mgh$$
$$= mgl\cos 30° = \frac{\sqrt{3}}{2}mgl$$
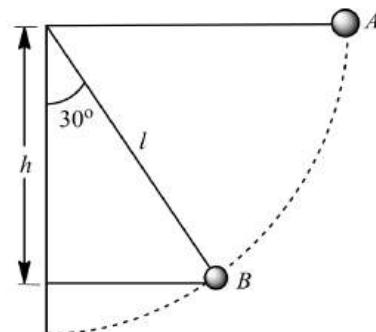
Kinetic energy gained = loss of potential energy

$$= \frac{\sqrt{3}}{2}mgl$$

115 (a)

In this case,

$$\text{Stress} = \frac{mg}{A}$$
$$\text{Strain} = \frac{l}{L} \qquad \text{(where } l \text{ is extension)}$$

Now, Young's modulus Y is given by

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{mg/A}{l/L}$$
$$mg = \frac{YAl}{L}$$

So, $\qquad kl = \frac{YAl}{L} \qquad (\because \ mg = kl)$

$\qquad (k \text{ is force constant})$

Now, frequency is given by

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$= \frac{1}{2\pi}\sqrt{\left(\frac{YA}{mL}\right)}$$

116 (d)

The time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$
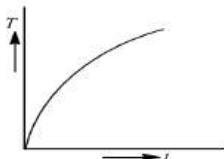
Here, l is the length of the pendulum.
On squaring both sides

$$T^2 = \frac{4\pi^2 l}{g}$$

$$\Rightarrow \qquad T^2 \propto l$$

So, the graph between time period T and length l of the pendulum is a parabola.

117 (c)

Let O be the mean position and x be the distance of coin from O. The coin will slip if centrifugal force on coin just becomes equal to force of friction ie, $mx\omega^2 = \mu mg$

The coin will slip if $x = $ maximum = amplitude A

∴ $mA\omega^2 = \mu mg$ or $A = \mu g/\omega^2$

118 (a)

A harmonic oscillation of constant amplitude and of single frequency is called simple harmonic oscillation.

Here, $\qquad x = 8 \sin \omega t + 6 \cos \omega t$

So, $\qquad a_1 = 8$ cm and $a_2 = 6$ cm

∴ Amplitude of motion

$$A = \sqrt{a_1^2 + a_2^2}$$
$$= \sqrt{8^2 + 6^2}$$
$$= \sqrt{64 + 36} = \sqrt{100} = 10 \text{ cm}$$

119 (c)

Resultant amplitude $= \sqrt{3^2 + 4^2} = 5$

120 (a)

In order to find the time taken by the particle from -12.5 cm to +12.5 cm on either side of mean position, we will find the time taken by particle to go from $x = -12.5$ cm to $x = 0$ and to go from $x = 0$ to $x = +12.5$ cm.

Let the equation of motion be $x = A \sin \omega t$

First, the particle moves from $x = -12.5$ cm to $x = 0$

∴ $12.5 = 25 \sin \omega t$ ∵ $A = 25$ cm

$\Rightarrow \qquad \frac{1}{2} = \sin \omega t$

$\Rightarrow \qquad \omega t = \frac{\pi}{6}$

∴ $\qquad t = \frac{\pi}{6\omega}$

Similarly to go from $x = 0$ to $x = 12.5$ cm

$$\omega t = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{6\omega}$$

∴ Total time taken from $x = -12.5$ cm to $x = 12.5$ cm

$$t' = \frac{\pi}{6\omega} + \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$$
$$= \frac{\pi}{3(\frac{2\pi}{T})} = \frac{T}{6} = \frac{3}{6} = 0.5 \text{ s}$$

121 (c)

$x(t) = A \cos(\omega t + \phi)$ ...(i)

∴ $\quad 1 = A \cos(\pi \times 0 + \phi) = A \cos \phi$ ...(ii)

Velocity$= \frac{d[x(t)]}{dt} = -A\omega \sin(\omega t + \phi)$

$\pi = -A \times \pi \sin(0 + \phi) = -\pi A \sin \phi - 1 = A \sin \phi$ ...(iii)

Squaring and adding Eqs. (ii) and (iii), we have

$1 + 1 = A^2(\cos^2 \phi + \sin^2 \phi) = A^2$ or $A = \sqrt{2}$ cm

122 (b)

The two springs are in series. Therefore, the time period is

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$= 2\pi\sqrt{m\left(\frac{k_1 + k_2}{k_1 k_2}\right)}$$

As $m = 16$ kg;

$$T = 8\pi\sqrt{\frac{k_1 + k_2}{k_1 k_2}}$$

123 (a)

$a = 10 \times 10^{-2} m$ and $\omega = 10 \ rad/s$

$A_{\max} = \omega^2 a = 10 \times 10^{-2} \times 10^2 = 10 \ m/s^2$

124 (d)

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_P}{T_e} = \sqrt{\frac{g_e}{g_P}} = \sqrt{\frac{2}{1}} \Rightarrow T_P$$
$$= \sqrt{2}T_e$$

125 (a)

For a particle executing SHM

$$v_{\max} = A\omega$$
$$= 4 \times \frac{2\pi}{T} = 4 \times \frac{2\pi}{8}$$
$$= \pi \text{ cms}^{-1}$$

126 (c)

The potential energy of a particle executing SHM is periodic with time period $\frac{T}{2}$

127 (b)

Length of the line = Distance between extreme positions of oscillation = $4 \ cm$

So, Amplitude $a = 2 \ cm$

also $v_{\max} = 12 \ cm/s$

∵ $v_{\max} = \omega a = \frac{2\pi}{T} a$

$$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 2}{12} = 1.047\ s$$

128 (c)

Spring constant $(K) \propto \frac{1}{\text{Length of the spring } (l)}$

as length becomes half, k becomes twice is $2K$

129 (d)

KE=PE

$$\Rightarrow \quad \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2 y^2$$

$$\Rightarrow \qquad\qquad y = \frac{a}{\sqrt{2}}$$

130 (c)

$x = a\sin\omega t$

And $\ y = b\sin(\omega t + \pi) = -b\sin\omega t$

or $\ \frac{x}{a} = -\frac{y}{b}\ $ or $\ y = -\frac{b}{a}x$

it is an equation of a straight line.

131 (d)

In simple harmonic motion, energy changes from kinetic to potential and potential to kinetic but the sum of two always remains constant

132 (b)

$\frac{x}{a} = \sin\omega t$

$\frac{y}{a} = \cos\omega t$

$\frac{y^2}{a^2} + \frac{x^2}{a^2} = 1 \Rightarrow y^2 + x^2 = a^2$

133 (a)

$n = \frac{\omega}{2\pi} = \frac{220}{2\pi} = 35Hz$

$v_{max} = \omega a = 220 \times 0.30 m/s = 66m/s$

134 (b)

$v_1 = \frac{dy_1}{dt}$

$= 0.1 \times 100\pi \cos(100\pi t + \pi/3)$

$v_2 = \frac{dy_2}{dt} = 0.1 \times 100\pi \sin 100\ \pi t$

$= 0.1 \times 100\pi \cos(100\pi t + \pi/2)$

Phase difference between two velocities

$(100\pi t + \pi/2) - (100\pi t + \pi/3)$

$= \pi/2 - \pi/3 - \pi/6$

135 (b)

When the cylinder is given a small downward displacement, say y, the additional restoring force is due to (i) additional extension y, which is $F_1 = ky$ (ii) Additional buoyancy, which is $F_2 = A Y d\ g$

Total restoring force, $-F = F_1 + F_2 = (k + Adg)y$

=new force constant

$$\therefore n - \frac{1}{2\pi}\sqrt{\frac{k'}{k}} = \frac{1}{2\pi}\sqrt{\frac{k + Adg}{M}}$$

136 (b)

$T \propto \sqrt{l} \Rightarrow T^2 \propto l$
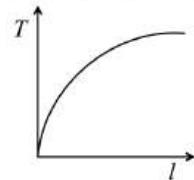
137 (c)

In forced oscillations, the body oscillates at the angular frequency of the driving force

138 (d)

$x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$

Acceleration, $a = \frac{d^2 x}{dt^2}$

$= -\omega^2 x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$

$= -\omega^2 x_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$

So, $\quad A = \omega^2 x_0$

and $\quad \delta = \frac{3\pi}{4}$

139 (a)

At the time $t = \frac{T}{4} = \frac{4}{4} = 1$ sec after passing from mean position, the body reaches at it's extreme position. At extreme, position velocity of body becomes zero

140 (b)

Acceleration, $a = -\omega^2 y = \frac{-4\pi^2}{T^2}y$

or $\ T = \left(\frac{4\pi^2 y}{a}\right)^{1/2} = 2\pi\sqrt{\frac{y}{a}}$

$= 2 \times \frac{22}{7} \times \sqrt{\frac{3}{12}} = 3.14$

141 (a)

The forces that act on the block are qE and mg. Since qE and mg are constant forces, the only variable elastic force changes by kx. Where x is the elongation in the spring $\Rightarrow$ unbalanced (restoring) force $= F = -kx$

$$\Rightarrow -m\omega^2 x = -kx \Rightarrow \omega = \sqrt{\frac{k}{M}} = T$$

142 (d)

Wave length = velocity of wave × Time period

$\lambda = 300 \times 0.05 \Rightarrow \lambda = 15$ metre

According to problem path difference between two points $= 15 - 10 = 5m$

$\therefore$ Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

$$= \frac{2\pi}{15} \times 5 = \frac{2\pi}{3}$$

143 (c)

$v_{\max} = a\omega \Rightarrow \omega = \frac{v_{\max}}{a} = \frac{10}{4}$

Now, $v = \omega\sqrt{a^2 - y^2} \Rightarrow v^2 = \omega^2(a^2 - y^2) \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2}$

$$\Rightarrow y = \sqrt{a^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}\ cm$$

144 (b)

From the relation of restitution $\frac{h_n}{h_0} = e^{2n}$ and

$h_n = h_0(1 - \cos 60°) \Rightarrow \frac{h_n}{h_0} = 1 - \cos 60°$

$$= \left(\frac{2}{\sqrt{5}}\right)^{2n}$$

$$\Rightarrow 1 - \frac{1}{2} = \left(\frac{4}{5}\right)^n \Rightarrow \frac{1}{2} = \left(\frac{4}{5}\right)^n$$

Taking log of both sides we get

$\log 1 - \log 2 = n(\log 4 - \log 5)$

$0 - 0.3010 = n(0.6020 - 0.6990)$

$-0.3010 = -n \times 0.097 \Rightarrow n = \frac{0.3010}{0.097} = 3.1 \approx 3$

145 (d)

Potential energy of particle

$$U = \frac{1}{2}m\omega^2 y^2$$

Potential energy of maximum particle

$$E = \frac{1}{2}m\omega^2 A^2$$

According to given position,

The potential energy $\quad U = \frac{E}{2}$

or $\quad \frac{1}{2}m\omega^2 y^2 = \frac{1}{2} \times \frac{1}{2}m\omega^2 A^2$

$$y^2 = \frac{A^2}{2}$$

$$y = \frac{A}{\sqrt{2}}$$

147 (b)

$x_1 = a\sin(\omega \times 1) = a\sin\omega$

and $\quad x_2 = a\sin(\omega \times 2) - a\sin\omega$

$\frac{x_2}{x_1} = \frac{\sin(2\omega) - \sin\omega}{\sin\omega}$

$\quad = \sin 2 \times (2\pi/8) - \sin 2\pi/8$

$\quad = \frac{1 - (1/\sqrt{2})}{(1/\sqrt{2})} = \frac{\sqrt{2} - 1}{1}$

Or $\frac{x_1}{x_2} = \frac{1}{\sqrt{2} - 1} = \frac{\sqrt{2} + 1}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$

$$= \frac{\sqrt{2} + 1}{2 - 1}$$

$$= 2.414 = 2.4$$

148 (b)

$T' = 2\pi\sqrt{l/(g/6)} = \sqrt{6}T$ Hence, the clock will tick in one minute

$= 60/\sqrt{6} = 24.5$ times

149 (d)

Maximum velocity $= a\omega = a\sqrt{\frac{K}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$

150 (b)

Let the minimum amplitude of SHM is a.

Restoring force on spring

$\quad F = ka$

Restoring force is balanced by weight mg of block. For mass to execute simple harmonic motion of amplitude a.

$\quad ka = mg$

or $\quad a = \frac{mg}{k}$

Here, $m = 2\ \text{kg}, k = 200\ \text{Nm}^{-1}, g = 10\ \text{ms}^{-2}$

$\therefore \quad a = \frac{2 \times 10}{200} = \frac{10}{100}\ \text{m}$

$$= \frac{10}{100} \times 100\ \text{cm} = 10\ \text{cm}$$

Hence, minimum amplitude of the motion should be 10 cm, so that the mass gets detached from the pan.

151 (d)

The relation between time period (T) and length of pendulum (l) is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

On squaring and rearranging the terms, we get

$$T^2 = 4\pi^2\frac{l}{g}$$

Which is general equation $(y^2 = 4ax)$ of a parabola.

153 (c)

$$\frac{T_1}{T} = \sqrt{\frac{l_1}{l}} = \sqrt{\frac{16}{1}} = 4$$

Or $t_1 = 4T$

Let after time t, the pendulum be in the same phase. It will be so then

$$\frac{t}{T_1} = \frac{t}{T} - 1 = \frac{t - T}{T}$$

or $\frac{t}{4T} = \frac{t-T}{T}$

or $t = 4t - 4t$

or $3t = 4T$

or $t = 4T/3$

154 (c)

$a = -\omega^2 x \Rightarrow \left|\frac{a}{x}\right| = \omega^2$

155 (b)

The particle will meet at the mean position when P completes one oscillation and Q completes half an oscillation

So $\frac{v_P}{v_Q} = \frac{a\omega_P}{a\omega_Q} = \frac{T_Q}{T_P} = \frac{6}{3} = \frac{2}{1}$

156 (c)

Using, $T = 2\pi\sqrt{\frac{M}{k}}$, we have

$1 = 2\pi\sqrt{\frac{200}{k}}$ ...(i)

and $0.5 = 2\pi\sqrt{\frac{(200-m)}{k}}$ ...(ii)

on solving, $m = 150$ g.

157 (c)

For a simple pendulum time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or $$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

∴ $$\omega = \sqrt{\frac{g}{l}}$$

$$\omega^2 = \frac{g}{l}$$

...(i)

Amplitude, when angular displacement is 60°

$$= \frac{2\pi l}{360} \times 60 = \frac{2\pi l}{6}$$

Therefore, displacement when angular displacement is 30°

$$= \frac{1}{2}\left(\frac{2\pi l}{6}\right)$$

$$y = \frac{\pi l}{6}$$

...(ii)

Acceleration $(\alpha) = -\omega^2 y$

Using Eqs. (i) and (ii), we get

$$\alpha = -\frac{g}{l} \times \frac{\pi l}{6} = -\frac{10 \times 3.14}{6}$$

$$= -5.2 \text{ ms}^{-2} = -5 \text{ ms}^{-2}$$

158 (b)
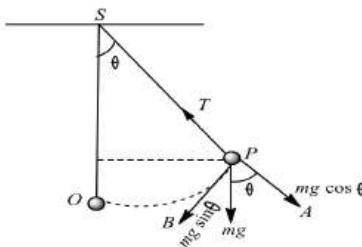
When the bob is displaced to position P, through a small angle θ from the vertical, the various forces acting on the bob at P are

(i) the weight mg of the bob acting vertically downwards

(ii) the tension T in the string acting along PS

Resolving mg into two rectangular components, we get

(a) $mg \cos θ$ acts along PA, opposite to tensions, we get

(b) $mg \sin θ$ acts along PB, tangent to the arc OP and directed towards O.

If the string neither slackens nor breaks but remains taut, then

$$T = mg \cos θ$$

The force $mg \sin θ$ tends to bring the bob back to its mean position O.

∴ Restoring force acting on the bob is

$$F = -mg \sin θ$$

159 (a)

At $x = 0$, kinetic energy is maximum and potential energy is minimum.

160 (d)

Minimum velocity is zero at the extreme positions. After 1s it will be at extreme position

161 (d)

From the given equation $\omega = 2\pi n = 4\pi \Rightarrow n = 2 Hz$

162 (a)

If a particle executes SHM, its kinetic energy is given by

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - x^2)$$

or $$\text{KE} = \frac{1}{2}k(A^2 - x^2)$$

where $k = m\omega^2 = $ constant

Its potential energy is given by

$$\text{KE} = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}kx^2$$

Thus, total energy of particle

$$E = \text{KE} + \text{PE}$$

$$= \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}kx^2$$

$$= \frac{1}{2} kA^2$$

Hence, $PE = \frac{1}{2} kx^2 = \frac{1}{2} k \left(\frac{A}{2}\right)^2$

$(\because \; x = \frac{A}{2})$

$$= \frac{1}{4} \left(\frac{1}{2} kA^2\right)$$

$$= \frac{1}{4} E$$

Hence, potential energy is one-fourth of total energy.

163 (b)

The effective acceleration of a bob in water
$= g' = g \left(1 - \frac{\sigma}{\rho}\right)$ where σ and ρ are the density of water and the bob respectively. Since the period of oscillation of the bob in air and water are given as

$T = 2\pi \sqrt{\frac{l}{g}}$ and $T' = 2\pi \sqrt{\frac{l}{g'}}$

$\therefore \frac{T}{T'} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g(1 - \sigma/\rho)}{g}} = \sqrt{1 - \frac{\sigma}{\rho}} = \sqrt{1 - \frac{1}{\rho}}$

Putting $\frac{T}{T'} = \frac{1}{\sqrt{2}}$. We obtain, $\frac{1}{2} = 1 - \frac{1}{\rho} \Rightarrow \rho = 2$

164 (d)

Time period of simple pendulum is given by

$$T_1 = 2\pi \sqrt{\frac{l}{g}}$$

and time period of uniform rod in given position is given by

$$T_2 = 2\pi \sqrt{\frac{\text{Inertia factor}}{\text{spring factor}}}$$
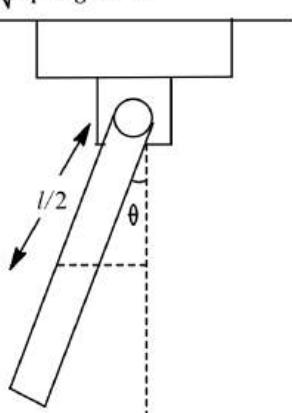
Here, inertia factor=moment of inertia of rod at one end

$$= \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

Spring factor=restoring torque per unit angular displacement

$$= mg \times \frac{l}{2} \frac{\sin\theta}{\theta}$$

$$= mg \times \frac{l}{2} \qquad \text{(if } \theta$$

is small)

$\therefore \quad T_2 = 2\pi \sqrt{\frac{ml^2/3}{mgl/2}} = 2\pi \sqrt{\frac{2}{3} \frac{l}{g}}$

Hence, $\quad \frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

165 (b)

When the particle of mass m at O is pushed by y in the direction of A the spring A will be compressed by y while spring B and C will be stretched by $y' = y \cos 45°$. So that the total restoring force on the mass m along OA
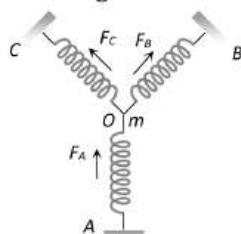
$F_{net} = F_A + F_B \cos 45° + F_C \cos 45°$
$= ky + 2ky' \cos 45°$
$$= ky$$
$$+ 2k(y \cos 45°) \cos 45° = 2ky$$

Also $F_{net} = k'y \Rightarrow k'y = 2ky \Rightarrow k' = 2k$

$T = 2\pi \sqrt{\frac{m}{k'}} = 2\pi \sqrt{\frac{m}{2k}}$

166 (b)

Total energy= PE + KE. When a particle executes SHM, there will be two position in each cycle where the PE is equal to KE of the body in SHM

167 (c)

$$R \sin \delta = 4$$

and $\quad R \cos \delta = 4$

$$R = 4\sqrt{2}$$

168 (a)

$v_0 = r\omega = 1$
And $a_0 = \omega^2 r = 1.57$
$\omega = \omega^2 r / r\omega = 1.57/1$
Or $2\pi v = 1.57$
$$v = \frac{1.57}{2 \times 3.14} = \frac{1}{4} = 0.25 \text{ s}^{-1}$$

169 (c)

$v_{max} = 100 = a\omega$;
$\omega = 100/a = 100/10 = 10 \text{ rads}^{-1}$.
$v^2 = \omega^2(a^2 - y^2)$
or $50^2 = 10^2(10^2 - y^2)$ or $25 = 100 - y^2$
or $y = \sqrt{75} = 5\sqrt{3}$ cm.

170 (d)

For the given figure $f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ ...(i)

If one spring is removed, then $k_{eq} = k$ and

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{... (ii)}$$

From equation (i) and (ii),

$\frac{f}{f'} = \sqrt{2} \Rightarrow f' = \frac{f}{\sqrt{2}}$

171 (b)

Energy of oscillation, $E = \alpha A^4$

KE of mass at is

$$K = E - U = \alpha(A^4 - x^4)$$
$$K = -3U$$
$$\alpha(A^4 - x^4) = 3\alpha x^4$$
$$x = \pm\frac{A}{\sqrt{2}}$$

172 (d)

$v_1 = \frac{dy_1}{dt} = 2 \times 10\cos(10t + \theta);$

$v_2 = -3 \times 10\sin 10t = 30\cos(10 + \pi/2)$

$\therefore$ Phase difference$= (10t + \theta) - (10t + \pi/2)$

173 (d)

$y = a\sin\omega t; v = \frac{dy}{dt} = a\omega\cos\omega t$

$= a\omega\sin(\omega t + \pi/2)$

Acceleration $A = \frac{dv}{dt} = -\omega^2 a\sin\omega t$

$= \omega^2 a\sin(\omega t + \pi)$

174 (b)

If at any instant displacement is y then it is given that

$U = \frac{1}{2} \times E \Rightarrow \frac{1}{2}m\omega^2 y^2 = \frac{1}{2} \times \left(\frac{1}{2}m\omega^2 a^2\right)$

$\Rightarrow y = \frac{a}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 4.2 \, cm$

175 (d)

$Y = 3\sin 31\,4t + 4\cos 31\,4t$

$= r\cos\theta\sin 314t + r\sin\theta\cos 31\,4t$

$= r\sin(314\,t + \theta)$ ...(i)

Where $r\cos\theta = 3$ and $r\sin\theta = 4$

$\therefore r^2(\cos^2\theta + \sin^2\theta) = 3^2 + 4^2 = 25$

or $r = 5 \, cm$

176 (a)

Because acceleration $\propto$ displacement

177 (d)

Equation of simple harmonic wave is

$$y = A\sin(\omega t + \emptyset)$$

Here, $y = \frac{A}{2}$

$\therefore \qquad A\sin(\omega t + \emptyset) = \frac{A}{2}$

So, $\delta = \omega t + \emptyset = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

So, the phase difference of the two particles when they are crossing each other at $y = \frac{A}{2}$ in opposite directions are

$$\delta = \delta_1 - \delta_2 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

178 (a)

$U = \frac{F^2}{2K} \Rightarrow U \propto \frac{1}{K} \Rightarrow \frac{U_1}{U_2} = \frac{K_2}{K_1} = 2$

179 (d)

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{4m}{m}} = 2 \Rightarrow T_2$$

$$= 2 \times 2 = 4s$$

180 (d)

$$T' = T\sqrt{\frac{l + 2l/100}{l}}$$

$$= T\left(1 + \frac{2}{100}\right)^{1/2} = 2\left(1 + \frac{1}{100}\right)$$

$\therefore \quad T' - T = \frac{2}{100} = \frac{1}{50}s$

Therefore, loss in seconds per day

$= \frac{1/50}{2} \times 24 \times 60 \times 60 = 864 \, s$

181 (c)

Given, mass of bob$=M$

Length of simple pendulum$=L$
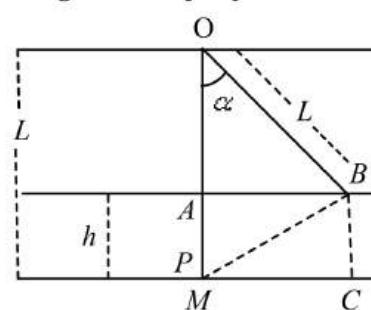
Now, in ΔOAB, $\cos\alpha = \frac{OA}{OB}$

or $\cos\alpha = \frac{OA}{L}$

or $AO = L\cos\alpha$

and $AP = OP - AO$

$AP = L - L\cos\alpha$

or $h = L(1 - \cos\alpha)$

The maximum kinetic energy of bob

$= $ Maximum potential energy

$KE = Mgh$

$KE = MgL(1 - \cos\alpha$

182 (a)

Velocity is same. So by using $v = a\omega$

$\Rightarrow A_1\omega_1 = A_2\omega_2 = A_3\omega_3$

183 (d)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T' = 2\pi\sqrt{\frac{2m}{2k}} = 2\pi\sqrt{\frac{m}{k}} = T$$

184 (b)

$F = kx \Rightarrow mg = kx \Rightarrow m \propto kx$

Hence $\dfrac{m_1}{m_2} = \dfrac{k_1}{k_2} \times \dfrac{x_1}{x_2} \Rightarrow \dfrac{4}{6} = \dfrac{k}{k/2} \times \dfrac{1}{x_2}$

$\Rightarrow x_2 = 3\ cm$

185 (c)

In series $k_{eq} = \dfrac{K_1 K_2}{K_1 + K_2}$ so time period $T = $

$2\pi\sqrt{\dfrac{m(K_1+K_2)}{K_1 K_2}}$

186 (a)

The object is not detached from the platform if

$$mg = m\omega^2 r = m\frac{4\pi^2}{T^2} \times r \ \ or \ \ T = 2\pi\sqrt{\frac{r}{g}}$$

187 (d)

Time period of a physical pendulum
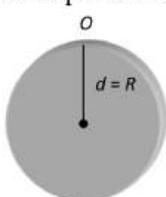
$$T = 2\pi\sqrt{\frac{I_0}{mgd}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}mR^2 + mR^2\right)}{mgR}}$$

$$= 2\pi\sqrt{\frac{3R}{2g}} \qquad ...(i)$$

$$T_{simple\ pendulum} = 2\pi\sqrt{\frac{l}{g}} \qquad ...(ii)$$

Equating (i) and (ii), $l = \dfrac{3}{2}R$

188 (d)

Here, $k = \dfrac{F}{x} = \dfrac{80}{0.2} = 400\ Nm^{-1}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.01}{400}}$$

$$= \frac{\pi}{100} = 0.03142\ s$$

190 (b)

Total mechanical energy in case of oscillation

$$E = \frac{1}{2}m\omega^2 A^2$$

$$\frac{E_1}{E_2} = \left[\frac{4}{8}\right]^2 = \frac{1}{4}$$

191 (a)

Potential energy of particle in SHM

$$U = \frac{1}{2}m\omega^2 x^2$$

or $\qquad U = \dfrac{1}{2}m(2\pi f)^2 x^2$

or $\qquad U = 2\pi^2 mf^2 x^2$

...(i)

Kinetic energy of particle in SHM

$$K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

or $\qquad K = 2\pi^2 mf^2(A^2 - x^2)$

...(ii)

Hence, total energy

$$E = K + U = 2\pi^2 mf^2 x^2 +$$

$2\pi^2 mf^2(A^2 - x^2)$

$$= 2\pi^2 mf^2 A^2 = \frac{2\pi^2 mA^2}{T^2}$$

$\left(\because \ T = \dfrac{1}{f}\right)$

Thus, it is obvious that total energy of particle executing simple harmonic motion depends on amplitude (A) and period (T).

192 (c)

In simple harmonic motion when a particle displaced to a position from its mean position then its kinetic energy gets converted in potential energy. Hence, total energy of particle remains constant or the total energy in simple harmonic motion does not depend is displacement x.

193 (a)

When the bob of pendulum is brought to a position making an angle θ with the equilibrium position, then height of fall of pendulum will be,

$h = l - l\cos\theta = l(1 - \cos\theta)$.

Taking free fall of the

$u = 0, a = g, g = h = l(1 - \cos\theta), v = ?$

Now, $v^2 = u^2 + 2gh = 0 + 2gl(1 - \cos\theta)$

or $\ \ v = \sqrt{2gl(1 - \cos\theta)}$

194 (b)

$\dfrac{mv^2}{r} = mg\sin\theta \ \ or \ \ v = \sqrt{gr\sin\theta}$

$= \sqrt{10 \times 200 \times \sin 15°} = 23\ ms^{-1}$

195 (b)
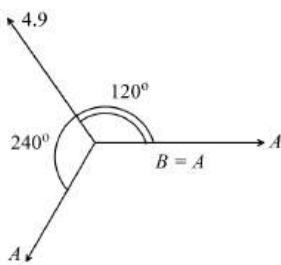
So $B = A, \phi = 240° = \frac{4\pi}{3}$

196 (b)

For series combination $K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$

$F = K_{eq} x \Rightarrow mg = \left(\frac{K_1 K_2}{K_1 + K_2}\right) x \Rightarrow x$

$= \frac{mg(K_1 + K_2)}{K_1 K_2}$

197 (b)

The wire may be treated as a spring for which force constant

$k_1 = \frac{\text{Force}}{\text{Extension}} = \frac{YA}{L} \left(\because Y = \frac{F}{A} \times \frac{L}{\Delta L}\right)$

Spring constant of the spring $k_2 = K$

Hence spring constant of the combination (series)

$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{(YA/L)K}{(YA/L) + K} = \frac{YAK}{YA + KL}$

$\therefore$ Time period $T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \left[\frac{(YA+KL)m}{YAK}\right]^{1/2}$

199 (d)

In vacuum, $T = 2\pi \sqrt{\frac{l}{g}}$
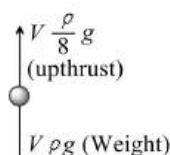
Let V be the volume and T be the density of the mass of the bob.

Net downward force acting on the bob in side the liquid = Weight − upthrust

$= V\rho g - V.\frac{T}{8}g = \frac{7}{8}V\rho g$

i.e., effective value of g is $\frac{7}{8}g$

So, time period of the bob inside the liquid

$\therefore T_l = 2\pi \sqrt{\frac{l}{(7/8)g}} = 2\pi \sqrt{\frac{l}{g}} \times \sqrt{\frac{8}{7}} = \sqrt{\frac{8}{7}}T$

200 (a)

Effective spring constant of parallel combination

$k_e = k_1 + k_2$

201 (c)

Maximum force= $m\omega^2 a = m4\pi v^2 a$

$= 1 \times 4\pi^2 \times (60)^2 \times 0.02 = 288\pi^2$

202 (a)

The maximum velocity of a particle performing SHM is given by $v = A\omega$, where A is the amplitude and ω is the angular frequency of oscillation.

$\therefore \qquad 4.4 = (7 \times 10^{-3}) \times 2\pi/T$

$\Rightarrow \quad T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01$ s

203 (b)

Maximum acceleration is given by

$= a\omega^2 = 24$ ms^{-2} ...(i)

Maximum velocity= $a\omega = 16$ ms^{-2} ...(ii)

Dividing Eq. (i) by Eq. (ii)

$\frac{a\omega^2}{a\omega} = \omega = \frac{24}{16} = \frac{6}{4} = \frac{3}{2}$

Now putting the value of ω in Eq. (ii), we get

$a \times \frac{3}{2} = 16$

$a = \frac{32}{3}$ m

204 (b)

$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} = \frac{\left(\frac{a}{2}\right)^2}{a^2} = \frac{1}{4} \Rightarrow U = \frac{E}{4}$

205 (b)

Here, $y_1 = \frac{1}{2}\sin \omega t + \frac{\sqrt{3}}{2}\cos \omega t$

$= \cos\frac{\pi}{3}\sin \omega t + \sin\frac{\pi}{2}\cos \omega t$

$\therefore \qquad y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$

Similarly, $\quad y_2 = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

$\therefore$ Phase difference $\Delta\emptyset = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

206 (c)

We have, $T \propto \sqrt{l}$,

$\therefore \frac{T_1}{T} = \sqrt{\frac{0.01l}{l}}$

$= \left(1 + \frac{1}{100}\right)^{1/2} = \left[1 + \frac{1}{2 \times 100}\right]$

$\therefore$ % increase in time period

$= \left(\frac{T_1 - T}{T}\right) \times 100$

$= \frac{1}{2 \times 100} \times 100 = 0.5\%$

207 (d)

Less damping force gives a taller and narrower resonance peak

Amplitude / Frequency
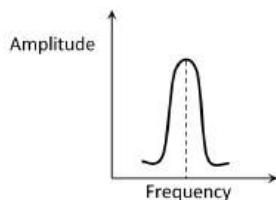
208 (d)

KE at the lowest position $= \frac{1}{2}mv^2$

$$= \frac{1}{2}m(3)^2 = \frac{9}{2}m$$

When the length makes an angle $\theta(= 60°)$ to the vertical, the bob of the pendulum will have both KE and PE. If v is the velocity of bob at this position and h is the height of the bob w.r.t. B, then total energy of the bob

$$= \frac{1}{2}mv^2 + mgh$$

But $h = l - l\cos\theta$

$$= l(1 - \cos\theta)$$
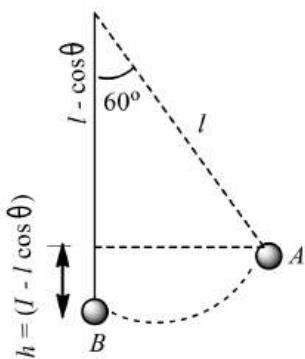
$$= 0.5(1 - \cos 60°) = 0.5\left(1 - \frac{1}{2}\right) = \frac{1}{4}$$

$$E = \frac{1}{2}mv^2 + m \times 10 \times \frac{1}{4}$$

$$= \frac{1}{2}mv^2 + \frac{5}{2}m$$

According to law of conservation of energy

$$\frac{1}{2}mv^2 + \frac{5m}{2} = \frac{9}{2}m$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{9}{2}m - \frac{5}{2}m = 2m$$

$$\therefore \qquad u = 2 \text{ ms}^{-1}$$

209 (b)

$T = 2\pi\sqrt{l/g}$;

$$\log T = \log 2 + \log \pi + \frac{1}{2}\log l - \frac{1}{2}\log g$$

Differentiating it we get

$$\frac{dT}{T} = \frac{1}{2}\frac{dl}{l} - \frac{1}{2}\frac{dg}{g} = -\frac{1}{2}\frac{dg}{g} \quad (\because l \text{ is constant})$$

% change in time period

$$= \frac{dT}{T} \times 100 = -\frac{1}{2}\frac{dg}{g} \times 100$$

$$= -\frac{1}{2}\left(\frac{-2}{100}\right) \times 100 = 1\% \text{ (increase)}$$

210 (b)

The spring-mass system oscillates in SHM, its time period is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

When spring is cut in ratio 1:3, the new time constant is $k' = 3k$

$$\therefore \qquad \frac{T}{T'} = \sqrt{\frac{3k}{k}}$$

$$\Rightarrow \qquad T' = \frac{T}{\sqrt{3}}$$

211 (a)

System is equivalent to parallel combination of springs

$$\therefore K_{eq} = K_1 + K_2 = 400 \text{ and}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{0.25}{400}} = \frac{\pi}{20}$$

213 (c)

The average acceleration of a particle performing SHM over one complete oscillation is zero.

216 (a)

It is required to calculate the time from extreme position

Hence, in this case equation for displacement of particle can be written as $x = a\sin\left(\omega t + \frac{\pi}{2}\right) = a\cos\omega t$

$$\Rightarrow \frac{a}{2} = a\cos\omega t \Rightarrow \omega t = \frac{\pi}{3} \Rightarrow \frac{2\pi}{T}.t = \frac{\pi}{3} \Rightarrow t = \frac{T}{6}$$

217 (a)

$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$, hence if l made 9 times T becomes 3 times.

Also time period of simple pendulum does not depend on the mass of the bob

218 (b)

Let T_1 and T_2 be the time period of the two pendulums $T_1 = 2\pi\sqrt{\frac{100}{g}}$ and $T_2 = 2\pi\sqrt{\frac{121}{g}}$

$[T_1 < T_2$ because $l_1 < l_2]$

Let at $t = 0$, they start swinging together. Since their time periods are different, the swinging will not be in unison always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together. Let longer length pendulum complete n oscillation and shorter length pendulum complete $(n + 1)$

oscillation, for the unision swinging, then
$(n + 1)T_1 = nT_2$

$(n + 1) \times 2\pi \sqrt{\dfrac{100}{g}} = n \times 2\pi \sqrt{\dfrac{121}{g}} \Rightarrow n = 10$

220 (c)

$Kx = mg \Rightarrow \dfrac{m}{K} = \dfrac{x}{g}$

So $T = 2\pi \sqrt{\dfrac{m}{K}} = 2\pi \sqrt{\dfrac{x}{9}} = 2\pi \sqrt{\dfrac{0.2}{9.8}} = \dfrac{2\pi}{7} s$

221 (a)

The acceleration of the vehicle down the plane=
$g \sin \alpha$.
The reaction force acting on the bob of pendulum
gives it an acceleration $a (= g \sin \alpha)$ up the plane.
This acceleration has two rectangular
components,
$a_x = a \cos \alpha = g \sin \alpha \cos \alpha$
And $a_y = a \sin \alpha = g \sin^2 \alpha$ as shown in figure.
The effective acceleration due to gravity acting on
the bob is given by
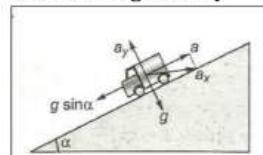
$g_{eff}^2 = a_x^2 + (g - a_y)^2 = a_x^2 g^2 + a_y^2 - 2ga_y$
$= g^2 \sin^2 \alpha \cos^2 \alpha + g^2 \sin^4 \alpha - 2g \times g \sin^2 \alpha$
$= g^2 \sin^2 \alpha (\cos^2 \alpha + \sin^2 \alpha) + g^2 - 2g^2 \sin^2 \alpha$
$= g^2 \sin^2 \alpha + g^2 - 2g^2 \sin^2 \alpha = g^2 (1 - \sin^2 \alpha)$
$= g^2 \cos^2 \alpha$
$\therefore \quad g_{eff} = g \cos \alpha$

Now, $T' = 2\pi \sqrt{\dfrac{L}{e_{eff}}}$

$= 2\pi \sqrt{\dfrac{L}{g \cos \alpha}}$

222 (d)

For SHM, $\dfrac{d^2 y}{dt^2} \propto -y$

223 (b)

Torque acting on the bob
$= I\alpha = -(mg)l \sin \theta$
Or $(m_i l^2)\alpha = -(m_g g)l\theta$
Or $\alpha = -\left(\dfrac{m_g g}{m_i l}\right)\theta = -\omega^2 \theta;$
Where $\omega^2 = \dfrac{m_g g}{m_i l}$

$\therefore T = \dfrac{2\pi}{\omega} = 2\pi \sqrt{\dfrac{m_i l}{m_g g}}$

224 (d)

The energy of simple harmonic oscillator

$$E = \dfrac{1}{2}m\omega^2 A^2$$

or $\quad E \propto A^2$

ie, energy is proportional to square of the
amplitude.

225 (c)

$y = 0.2 \sin(10\pi t + 1.5\pi) \cos(10\pi t + 1.5\pi)$
$= 0.1 \sin 2(10\pi t + 1.5\pi) \quad [\because \sin 2A$
$\qquad\qquad = 2 \sin A \cos A]$
$= 0.1 \sin(20\pi t + 3.0\pi)$

$\therefore$ Time period, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{20\pi} = \dfrac{1}{10} = 0.1 \, sec$

226 (b)

Two springs each of spring constant k_1 in parallel,
given equaivalent spring constant of $2k_1$ and this
is in series with spring of constant k_2, so
equivalent spring constant,

$k = \left(\dfrac{1}{k_2} + \dfrac{1}{2k_1}\right)^{-1}$

227 (b)

For displacement $OQ = 40$ cm; let t_1 be the time
taken then
$40 = 41 \sin \dfrac{2\pi}{24} t_1$, on solving $t_1 = 5.16$ s
For displacement $OQ = -9$ cm, let t_2 be the time
taken then $9 = 41 \sin \dfrac{2\pi}{12} t_2$,
On solving $t_2 = 0.84$ s
Total time $= 5.16 + 0.84 = 6.00$ s

229 (d)

The time period (T) of a simple pendulum
length l, is given by

$$T = 2\pi \sqrt{\dfrac{l}{g}} = \dfrac{1}{\text{frequency}(n)}$$

where g is acceleration due to gravity.

$\therefore \qquad \dfrac{n_1}{n_2} = \sqrt{\dfrac{l_2}{l_1}}$

$\Rightarrow \qquad \dfrac{l_1}{l_2} = \left(\dfrac{n_2}{n_1}\right)^2$

Given, $\quad \dfrac{n_2}{n_1} = \dfrac{3}{2}$

$\therefore \qquad \dfrac{l_1}{l_2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

230 (a)

It is a system of two springs in parallel. The
restoring force on the body is due to springs and
not due to gravity pull. Therefore slope is
irrelevant. Here the effective spring constant=
$k + k = 2k$
Thus time period, $T = 2\pi \sqrt{M/2k}$

231 (b)

Here $a = 12$ cms^{-2}, $x = 3$ cm

In SHM, acceleration $a = -\omega^2 x$

∴ Magnitude of acceleration $a = \omega^2 x$

(discarding off –ve sign)

∴ $\qquad \omega^2 = \dfrac{a}{x}$

or $\qquad \omega = \sqrt{\dfrac{a}{x}}$

or $\qquad \dfrac{2\pi}{T} = \omega = \sqrt{\dfrac{a}{x}}$

or $\qquad T = 2\pi\sqrt{\dfrac{a}{x}}$

$\qquad\qquad = 2\pi\sqrt{\dfrac{3}{12}}$

$\qquad\qquad = \pi s = 3.14$ s

232 (d)

In S.H.M. $v = \omega\sqrt{a^2 - y^2}$ and $a = -\omega^2 y$ when $y = 0$

$\Rightarrow v_{max} = a\omega$ and $a_{min} = 0$

233 (b)

Body collides elastically with walls of room. So, there will be no loss in its energy and it will remain colliding with walls of room after a regular time interval, so it's motion will be periodic. Since acceleration is not proportional to displacement, so it's motion is not SHM

234 (d)

$y = A\sin\left(\dfrac{2\pi}{T}\right)t$

$\Rightarrow \qquad \dfrac{A}{2} = A\sin\left(\dfrac{2\pi}{T}\right)t$

$\qquad \dfrac{\pi t}{2} = \dfrac{\pi}{6}$

$\qquad t = \dfrac{1}{3}$ s

235 (b)

Let k be the force constant of the shorter part of the spring of length $l/3$. In a complete spring, three springs are in series each of force constant k

$k_1 = k/2 = \dfrac{3k}{2}$

∴ $\dfrac{k}{k_1} = \dfrac{3K}{3K/2} = 2$ or $k : k_1 = 2 : 1$

236 (c)

In S.H.M. frequency of K.E. and P.E.

= 2 × (Frequency of oscillating particle)

237 (a)

$T = 2 = 2\pi\sqrt{\dfrac{M}{k}}$

and $2 + 1 = 2\pi\sqrt{\dfrac{M+4}{k}}$

or $3 = 2\pi\sqrt{\dfrac{k+4}{k}}$ so $\dfrac{4}{9} = \dfrac{M}{M+4}$

or $4M + 16 = 9M$ or $M = \dfrac{16}{5} = 3.2$ kg

238 (a)

Given $K.E. = P.E. \Rightarrow \dfrac{1}{2}mv^2 = \dfrac{1}{2}kx^2$

$\Rightarrow \dfrac{1}{2}m\omega^2(a^2 - x^2) = \dfrac{1}{2}m\omega^2 x^2$

$\Rightarrow a^2 - x^2 = x^2 \Rightarrow x^2 = \dfrac{a^2}{2} \Rightarrow \dfrac{x}{a} = \dfrac{1}{\sqrt{2}}$

239 (b)

Acceleration in SHM is directly proportional to displacement and is always directed to its mean position

240 (a)

In S.H.M. when acceleration is negative maximum or positive maximum, the velocity is zero so kinetic energy is also zero. Similarly for zero acceleration, velocity is maximum so kinetic energy is also maximum

241 (b)

The potential energy, $U = \dfrac{1}{2}kx^2$

$\qquad\qquad 2U = kx^2$

$\qquad\qquad 2U = -Fx$

$(\because F = -kx)$

or $\qquad \dfrac{2U}{F} = -x$

or $\qquad \dfrac{2U}{F} + x = 0$

242 (d)

$y = a\sin(\omega t + \phi_0)$. According to the equation

$y = \dfrac{a}{2} \Rightarrow \dfrac{a}{2} = a\sin(\omega t + \phi_0) \Rightarrow (\omega t + \phi_0) = \phi$

$\qquad\qquad = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

Physical meaning of $\phi = \dfrac{\pi}{6}$: Particle is at point P and it is going towards B
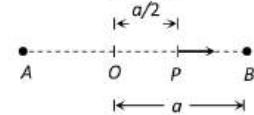
Physical meaning of $\phi = \dfrac{5\pi}{6}$: Particle is at point P and it is going towards O
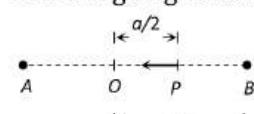
So phase difference $\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120°$

243 (c)

Velocity = acceleration
$$\omega\sqrt{a^2 - y^2} = \omega^2 y$$
$$\sqrt{(2)^2 - (1)^2} = \omega(1)$$
$$\Rightarrow \quad \omega = \sqrt{3}$$
$$T = \frac{2\pi}{\omega}$$
$$\Rightarrow \quad T = \frac{2\pi}{\sqrt{3}}$$

244 (c)

If amplitude is large motion will not remain simple harmonic

245 (d)

$$T = 2\pi\sqrt{\frac{l}{g}};$$
$$T = 2\pi\sqrt{\frac{1}{g + g/3}} = 2\pi\sqrt{\frac{3l}{4g}} = \left(\sqrt{\frac{3}{4}}\right)T'$$
or $t' = \frac{2T}{\sqrt{3}}$

246 (c)

$y = A\sin PT + B\cos PT$
Let $A = r\cos\theta$, $B = r\sin\theta$
$\Rightarrow y = r\sin(PT + \theta)$ which is the equation of SHM

248 (a)

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{3}{2} = \sqrt{\frac{m+2}{m}} \Rightarrow \frac{9}{4}$$
$$= \frac{m+2}{m}$$
$$\Rightarrow m = \frac{8}{5}kg = 1.6 \ kg$$

249 (b)

$$v_{max} = a\omega = a \times \frac{2\pi}{T} = (50 \times 10^{-3}) \times \frac{2\pi}{2}$$
$$= 0.15 \ ms^{-1}$$

250 (d)

The displacement of particle, executing SHM,
$$y = 5\sin\left(4t + \frac{\pi}{3}\right) \qquad \text{...(i)}$$
Velocity of particle, $\frac{dy}{dt} = \frac{5d}{dt}\sin\left(4t + \frac{\pi}{3}\right)$
$$= 5\cos\left(4t + \frac{\pi}{3}\right)4 = 20\cos\left(4t + \frac{\pi}{3}\right)$$
Velocity at $t = \left(\frac{T}{4}\right)$
$$\left(\frac{dy}{dt}\right)_{t=\frac{T}{4}} = 20\cos\left(4 \times \frac{T}{4} + \frac{\pi}{3}\right)$$
$$\Rightarrow u = 20\cos\left(T + \frac{\pi}{3}\right) \qquad \text{...(ii)}$$

Comparing the given equation with standard equation of SHM $y = a\sin(\omega t + \phi)$,
we get $\omega = 4$
As $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{4} \Rightarrow T = \left(\frac{\pi}{2}\right)$
Now, putting value of T in Eq. (ii), we get
$$u = 20\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = -20\sin\frac{\pi}{3}$$
$$= -20 \times \frac{\sqrt{3}}{2} = -10 \times \sqrt{3}$$
The kinetic energy of particle,
$$KE = \frac{1}{2}mu^2$$
$$\because m = 2g = 2 \times 10^{-3}kg$$
$$= \frac{1}{2} \times 2 \times 10^{-3} \times \left(-10\sqrt{3}\right)^2$$
$$= 10^{-3} \times 100 \times 3 = 3 \times 10^{-1} \Rightarrow K.E. = 0.3J$$

251 (a)

In first case, springs are connected in parallel, so their equivalent spring constant
$$k_p = k_1 + k_2$$
So, frequency of this spring-block system is
$$f_p = \frac{1}{2\pi}\sqrt{\frac{k_p}{m}}$$
or $\quad f_p = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$
but $\quad k_1 = k_2 = k$
$\therefore \quad f_p = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$
..(i)
Now in second case, springs are connected in series, so their equivalent spring constant
$$k = \frac{k_1 k_2}{k_1 + k_2}$$
Hence, frequency of this arrangement is given by
$$f_s = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}$$
or $\quad f_s = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$
...(ii)
Dividing Eq. (ii) by Eq. (i), we get
$$\frac{f_s}{f_p} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{2m}}}{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}} = \sqrt{\frac{1}{4}}$$
or $\quad \frac{f_s}{f_p} = \frac{1}{2}$

252 (b)

Using $F = kx \Rightarrow 10g = k \times 0.25 \Rightarrow k = \frac{10g}{0.25} = 98 \times 4$

Now $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2}{4\pi^2}k$

$\Rightarrow m = \frac{\pi^2}{100} \times \frac{1}{4\pi^2} \times 98 \times 4 = 0.98 \, kg$

253 (c)

$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow n \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$

$\Rightarrow \frac{n}{n_2} = \sqrt{\frac{4m}{m}} \Rightarrow n_2 = \frac{n}{2}$

254 (c)

When the displacement of bob is less than maximum, there will two compounding acceleratins $\vec{a_l}$ and $\vec{a_c}$ of the bob as shown in figure. Their resultant acceleration $\vec{a}$ will be represented by the diagonal of the parallelogram
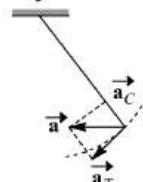
255 (b)

$K = \frac{1}{2}m\omega^2(a^2 - y^2)$

$\quad \frac{3}{4}E = \frac{1}{2}m\omega^2(a^2 - y^2)$

$\quad \frac{3}{4}\left(\frac{1}{2}m\omega^2 a^2\right) = \frac{1}{2}m\omega^2(a^2 - y^2)$

$\quad y^2 = a^2 - \frac{3}{4}a^2 = \frac{a^2}{4}$

$\Rightarrow \qquad y = \frac{a}{2}$

256 (c)

$T = 2\pi\sqrt{\frac{l}{g}}$. When lift is accelerated upwards with acceleration$a(= g/4)$. Then effective acceleration due to gravity inside the lift

$g_1 = g + a = g + \frac{g}{4} = \frac{5g}{4}$

$\therefore \quad T_1 = 2\pi\sqrt{\frac{l}{5g/4}} = 2\pi\sqrt{\frac{l}{g}} \times \frac{2}{\sqrt{5}} = \frac{2T}{\sqrt{5}}$

257 (c)

The total time from A to C

$t_{AC} = t_{AB} + t_{BC}$

$= (T/4) + t_{BC}$

Where T =time period of oscillation of spring mass system

t_{BC} can be obtained from, $BC = AB \sin(2\pi/T) t_{BC}$

Putting $\frac{BC}{AB} = \frac{1}{2}$ we obtain $t_{BC} = \frac{T}{12}$

$\Rightarrow \quad t_{AC} = \frac{T}{4} + \frac{T}{12} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

258 (b)

$\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T} \times T = \frac{4\pi^2}{T} = $ constant

260 (b)

Maximum force on body while in SMH
$= m\omega^2 a = 0.5 \times (2\pi/2)^2 \times 0.2 = 1 \, N$
Maximum force of friction$= \mu mg = 0.3 \times 0.5 \times 10 = 1.5 \, N$
Since the maximum force on the body due to SHM of the platform is less than the maximum possible frictional force, so the maximum force of friction will be equal to the maximum force acting on body due to SHM of platform *ie*, 1 N

262 (a)

Potential energy is minimum (in the case zero) at mean position ($x = 0$) and maximum at extreme positions ($x = \pm A$). At time $t = 0, x = A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x = 0$. Hence, this is also correct.

263 (b)

$v_{\max} = a\omega$ and
Maximum acceleration $= \omega^2 a$

$$= \left(\frac{v}{a}\right)^2 a = \frac{v^2}{a}$$

264 (b)

When bigger pendulum of time period ($5T/4$) completes one vibration, the smaller pendulum will complete ($5/4$) vibrations. It means the smaller pendulum will be leading the bigger pendulum by phase $T/4$ sec$= \pi/2$ rad $= 90°$

265 (b)

Time period of simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

From this formula it can be predicted that time period does not depend on the mass of bob.

266 (d)

$$T \propto \sqrt{l} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \Rightarrow \frac{2}{T_2} = \sqrt{\frac{l}{4l}} \Rightarrow T_2 = 4 \, s$$

267 (c)

$$y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3} \Rightarrow \frac{1}{2} = \sin\frac{2\pi t}{3}$$

$$\Rightarrow \sin\frac{2\pi t}{3} = \sin\frac{\pi}{6} \Rightarrow \frac{2\pi t}{3} = \frac{\pi}{6} \Rightarrow t = \frac{1}{4}\sec$$

268 (b)

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{or} \quad k = \frac{4\pi^2}{T^2}m = \omega^2 m$$

269 (b)

So $a = 6cm, \omega = 100 rad/s$

$$K_{\max} = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}\times 1 \times (100)^2 \times (6\times 10^{-2})^2$$
$$= 18\,J$$

270 (d)

As retardation $= bv$

$\therefore$ retarding force $= mbv$

$\therefore$ net restoring torque when angular displacement is θ is given by
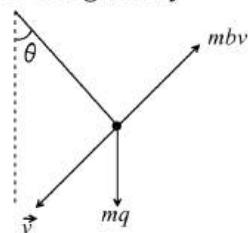
$$= -mg\ell\sin\theta + mbv\ell$$
$$\therefore I\alpha = -mg\ell\sin\theta + mbv\ell$$

where, $I = m\ell^2$

$$\therefore \frac{d^2\theta}{dt^2} = \alpha = -\frac{g}{\ell}\sin\theta + \frac{bv}{\ell}$$

for small damping, the solution of the above differential equation will be

$$\therefore \theta = \theta_0 e^{\frac{bt}{2}}\sin(\omega t + \phi)$$

$\therefore$ angular amplitude will be $= \theta.e^{\frac{-bt}{2}}$

According to question, in τ time (average life-time),

Angular amplitude drops to $\frac{1}{e}$ value of its original value (θ)

$$\therefore \frac{\theta_0}{e} = \theta_0 e^{-\frac{b\tau}{2}} \Rightarrow \frac{b\tau}{2} = 1$$

$$\therefore \tau = \frac{2}{b}$$

271 (c)

Effective value of acceleration due to gravity is zero in the satellite, $ie, g_{\text{eff}} = 0$. Hence, time period of pendulum

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{0}} = \infty$$

is infinite.

272 (d)

$$T \propto \frac{1}{\sqrt{k}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k}{4k}} = \frac{1}{2} \Rightarrow T_2 = \frac{T_1}{2}$$

273 (c)

The relation between acceleration (a) and displacement (x) for a body in SHM is

$$a = -\omega^2 x$$

Given, $a = -bx$

On comparing the two equations, we get

$$\omega^2 = b$$
$$\therefore \qquad \omega = \sqrt{b}$$

Since, $\omega = \frac{2\pi}{T}$

$$\therefore \qquad \frac{2\pi}{T} = \sqrt{b}$$
$$\Rightarrow \qquad T = \frac{2\pi}{\sqrt{b}}$$

274 (d)

The lift is moving with constant velocity so, there will be no change in the acceleration hence time period will remain same.

275 (a)

Here, Mass of the body, $m = 500g = 500 \times 10^{-3}kg$

Spring constant, $k = 8\pi^2$ N m^{-1}

The frequency of oscillation is

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{8\pi^2 \text{ N m}^{-1}}{500\times 10^{-3} \text{ kg}}} = 2 \text{ Hz}$$

276 (c)

Kinetic energy at mean position,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \sqrt{\frac{2K_{\max}}{m}}$$

$$= \sqrt{\frac{2\times 16}{0.32}} = \sqrt{100} = 10 m/s$$

277 (c)

In SHM, the total energy=potential energy + kinetic energy

or $E = U + K$
$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$= \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$$

where $k =$ force constant$= m\omega^2$

Thus, total energy depends on k and A.

278 (a)

$$n \propto \sqrt{\frac{k}{m}}$$

279 (d)

$$F_{\max} = m\omega^2 a = m\frac{4\pi^2}{T^2}a$$

$$= \frac{10}{1000} \times \frac{4 \times (\pi)^2}{(\pi/5)} \times 0.5 = 0.5 \text{ N}$$

280 (a)

Max, KE= Max. PE

$$= \frac{1}{2}mv^2 = \frac{1}{2}kx^2 = \frac{1}{2} \times 65 \times (0.11)^2$$

or $v^2 = \frac{65 \times (0.11)^2}{650 \times 10^{-3}}$ or $v = 1.1 \text{ ms}^{-1}$

281 (b)

$T \propto \sqrt{l}, \because$ effective length $l_{\text{sitting}} > l_{\text{standing}}$

282 (a)

Let the equations of two mutually perpendicular SHM's same frequency be

$$x = a_1 \sin \omega t \text{ and } y = a_2 \sin(\omega t + \emptyset)$$

Then, the general equation of Lissajous figure can be obtained

as $\quad \frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2} \cos \emptyset = \sin^2 \emptyset$

For $\qquad \emptyset = 0°: \frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2} = 0$

$\Rightarrow \left[\frac{x}{a_1} - \frac{y}{a_2}\right]^2 = 0 \Rightarrow \frac{x}{a_1} = \frac{y}{a_2} \Rightarrow y = \frac{a_2}{a_1}x$

This is an equation of a straight line passing through origin.

283 (a)

In this case springs are in parallel, so $k_{eq} = k_1 + k_2$

and $\omega = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$

284 (c)

$x = A \sin(\omega + \pi/2) = A \cos \omega t$

$\therefore \cos \omega t = x/A$ and $\sin \omega t = \sqrt{1 - (x^2/A^2)}$

$y = A \sin 2\omega t = \sqrt{1 - (x^2/\Delta A^2)}$

$y = A \sin 2\omega t = 2A \sin \omega t \cos \omega t$

or $y^2 = 4 A^2 \sin^2 \omega t \cos^2 \omega t$

$= 4A^2 \times \frac{x^2}{A^2} \times \left(\frac{A^2 - x^2}{A^2}\right) = 4x^2 \left(1 - \frac{x^2}{A^2}\right)$

285 (d)

The time period of oscillation of a spring does not depend on gravity

286 (a)

The standard equation in SHM is

$$x = a \cos(\omega t + \emptyset)$$

...(i)

Where a is amplitude, ω the angular velocity and ($\emptyset$) the phase difference.

Also, $\omega = \frac{2\pi}{T}$ where T is periodic time.

So, Eq. (i) becomes

$$x = a \cos\left(\frac{2\pi t}{T} + \emptyset\right)$$

...(ii)

Given, equation is

$$x = 0.01 \cos\left(\frac{2\pi t}{2} + \frac{\pi}{4}\right)$$

...(iii)

Comparing Eq. (ii) with Eq. (iii), we get

$$\frac{2\pi t}{T} = \frac{2\pi t}{2}$$

$\Rightarrow \qquad T = 2s$

So, frequency $n = \frac{1}{T} = \frac{1}{2} = 0.5$ Hz

287 (a)

When lift falls freely effective acceleration and frequency of oscillations become zero

$g_{eff} = 0 \Rightarrow T' = \infty$, hence a frequency $= 0$

288 (d)

When the force F is applied to one side of block A, let the upper face of A be displaced through distance ΔL

Then
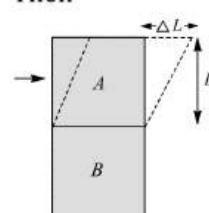
$\eta = \frac{F/L^2}{\Delta L/L}$ or $F = \eta L \Delta L \quad$ ...(i)

So, $F \propto \Delta L$ and this force is restoring one. So, if the force is removed, the block will execute SHM

From Eq. (i) spring factor $= \eta L$

Here, inertia factor$= M$

$\therefore$ Time period, $T = 2\pi \sqrt{\frac{M}{\eta L}}$

289 (d)

$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\left[\frac{g}{g\left(1 - \frac{1}{10}\right)}\right]} = \sqrt{\frac{10}{9}}$

$\Rightarrow \qquad T' = \sqrt{\frac{10}{9}} T$

290 (b)

$\frac{d^2 x}{dt^2} = -\alpha x \qquad\qquad$ ...(i)

We know,

$$a = \frac{d^2 y}{dt^2} = -\omega^2 x \qquad\qquad \text{...(ii)}$$

From Eqs.(i) and (ii), we have

$$\omega^2 = \alpha$$
$$\omega = \sqrt{\alpha}$$

or $\quad \dfrac{2\pi}{T} = \sqrt{\alpha}$

$\therefore \quad T = \dfrac{2\pi}{\sqrt{\alpha}}$

291 (c)

Time period $\quad T = 2\pi\sqrt{\dfrac{m}{k}}$

$\because \quad mg = kx$

$\therefore \quad T = 2\pi\sqrt{\dfrac{x}{g}}$

$(0.5)^2 = 4\pi^2 \times \sqrt{\dfrac{x}{10}}$

$\dfrac{(0.5)^2 \times 9.8}{4 \times 3.14 \times 3.14} = x$

$x = 0.0621$ m

$x = 6.2$ cm

292 (a)

Given that, the time period of particle $A = T$ and the time period of particle $B = \dfrac{5T}{4}$

Hence, the time difference $(\Delta T) = \dfrac{5T}{4} - T$

$\Rightarrow \quad \Delta T = \dfrac{T}{4}$

...(i)

The relation between phase difference and time difference is

$\Delta\emptyset = \dfrac{2\pi}{T}\Delta T$

$\Delta\emptyset = \dfrac{2\pi}{T} \times \dfrac{T}{4}$

$\Rightarrow \quad \Delta\emptyset = \dfrac{\pi}{2}$

293 (c)

As X and Y have negligible mass, both the spring balances read the same force $8 kg$ or $8 kg$

295 (c)

If a spring of spring constant k is divided into n equal parts, the spring constant of each part becomes nk. So, effective spring constant

$k = k_1 + k_2$

$= 4k + 4k = 8k$

296 (c)
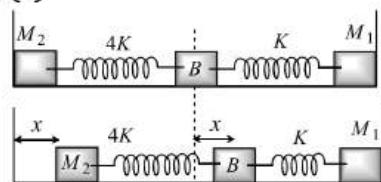
As springs and supports (M_1 and M_2) are having negligible mass. Whenever springs pull the massless supports, springs will be in natural

length. At maximum compression, velocity of B will be zero
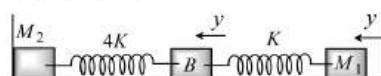
And by energy conservation

$\dfrac{1}{2}(4K)y^2 = \dfrac{1}{2}Kx^2 \Rightarrow \dfrac{y}{x} = \dfrac{1}{2}$

297 (a)

$A_{\max} = \omega^2 a$

299 (c)

Let displacement equation of particle executing SHM is

$y = a\sin\omega t$

As particle travels half of the amplitude from the equilibrium position, so $y = \dfrac{a}{2}$

Therefore, $\dfrac{a}{2} = a\sin\omega t \Rightarrow \sin\omega t = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$

$\Rightarrow \omega t = \dfrac{\pi}{6} \Rightarrow t = \dfrac{\pi}{6\omega} \Rightarrow t = \dfrac{\pi}{6\left(\dfrac{2\pi}{T}\right)}$ $\left(\text{As } \omega = \dfrac{2\pi}{T}\right)$

$\Rightarrow t = \dfrac{T}{12}$

Hence, the particle travels half of the amplitude from the equilibrium in $\dfrac{T}{12}s$

300 (d)

Let r be the amplitude of oscillation and T be the time period in SHM. Then total distance travelled in time $T = 4r$

$\therefore$ Average velocity, $v_{av} = \dfrac{4t}{T} = \dfrac{4r}{2\pi/\omega}$

$= \dfrac{2r\omega}{\pi} = \dfrac{2v_{\max}}{\pi}$

301 (a)

$\omega = \sqrt{\dfrac{\text{Acceleration}}{\text{Displacement}}} = \sqrt{\dfrac{2.0}{0.02}} = 10 \; rad \; s^{-1}$

302 (c)

$T = 2\pi\sqrt{\dfrac{l}{g_{net}}}$

$g_{net} = g + a = 10 + 10$

$g_{net} = 20 m/s^2$

$T' = \dfrac{T}{\sqrt{2}}$

303 (c)

$v_{\max} = \omega A \Rightarrow v = \dfrac{\omega A}{2} = \omega\sqrt{A^2 - y^2}$

$\Rightarrow A^2 - y^2 = \dfrac{A^2}{4} \Rightarrow y^2 = \dfrac{3A^2}{4} \Rightarrow y = \dfrac{\sqrt{3}A}{2}$

304 (b)

Amplitude of SHM of a particle $= X_M$
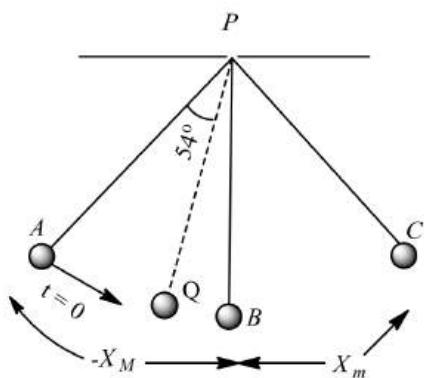
At $t = 0$ position of particle$=A$(given)

At any instant t displacement of particle$=y$ (say)

Angular velocity of particle$=\omega$

Then, $\quad y = X_M \sin \omega t$

$$y = X_M \sin \frac{2\pi t}{T}$$

At $t = 3.15\, T$

$$y = X_M \sin \frac{2\pi}{T}(3.15T)$$
$$= X_M \sin(2\pi \times 3.15)$$
$$y = X_M \sin(6\pi \times 54°)$$
$$= X_M \sin 54°$$
$$y = X_M \frac{(\sqrt{5}-1)}{4}$$

Since, the measurement starts from position A, then after $3.15\, T$ particle will be between O and X_m.

305 (a)

$$\frac{1}{2}m\omega^2(r^2 - y^2) = \frac{1}{3} \times \frac{1}{2}m\omega^2 r^2$$

or $\quad r^2 - y^2 = \frac{1}{3}r^2$

or $\quad 3r^2 - 3y^2 = r^2$

or $\quad 2r^2 - 3y^2 = 0$

or $\quad r = \sqrt{\frac{3}{2}} \times y = \sqrt{\frac{3}{2}} \times 4 = 2\sqrt{6}$ cm

306 (c)

$$x = 4(\cos \pi t + \sin \pi t)$$
$$= \frac{4}{\sqrt{2}} \times \sqrt{2}[\cos \pi t + \sin \pi t]$$
$$x = 4\sqrt{2} \sin[\pi t + \frac{\pi}{4}]$$

So, amplitude$=4\sqrt{2}$

307 (c)

From given equation $\omega = \frac{2\pi}{T} = 0.5\pi \Rightarrow T = 4s$

Time taken from mean position to the maximum displacement $= \frac{1}{4}T = 1\,s$

308 (b)

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000\,t$$

$\Rightarrow y = 2(1 + \cos t)\sin 1000\,t$

$\Rightarrow y = 2\sin 1000\,t + 2\cos t \sin 1000\,t$

$\Rightarrow y = 2\sin 1000\,t + \sin 999\,t + \sin 1001\,t$

It is a sum of three S.H.M.

309 (b)

$Mg\cos\theta$ will provide the required centripetal force,

So, $T - Mg\cos\theta = mv^2/L$

310 (d)

This is the special case of physical pendulum and in this case

$$T = 2\pi\sqrt{\frac{2l}{3g}}$$

$$\Rightarrow T = 2 \times 3.14\sqrt{\frac{2 \times 2}{3 \times 9.8}} = 2.31s \approx 2.4s$$

311 (c)

Displacement of particle in the case of SHM

$$y = A\sin(\omega t + \emptyset)$$

...(i)

$$y = 3\sin(0.2\,t)$$

...(ii)(given)

Comparing Eqs. (i) and (ii), we get

$$A = 3, \omega = 0.2$$

Now, particle distance $x = \frac{4}{3} = 1$

Kinetic energy in SHM

$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$= \frac{1}{2} \times 3 \times 10^{-3}(0.2)^2[3^2 - 1^2]$$
$$= 0.48 \times 10^{-3}\, J$$

312 (c)

Let the force constant of 2nd piece be k

As, $\quad k \propto \frac{1}{l}$

$\therefore \quad \frac{k_1}{k_2} = \frac{l_2}{l_1}$

or $\quad \frac{k}{k_2} = \frac{2l/3}{l}$

or $\quad k_2 = \frac{3k}{2}$

313 (b)

Let k be the spring constant of each half part of the spring. For a complete spring, the spring constant $k' = k/2$

(springs in series). When two splitted parts of a spring are connected to the body, then the springs are in parallel. Their effective spring constant,

$k' = k + k = 2k$

As $T = 2\pi\sqrt{\frac{m}{k}}$ or $T \propto \frac{1}{\sqrt{k}}$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2} \text{ or } T' = \frac{T}{2}$$

314 (c)

$$E = \frac{1}{2}m\omega^2 r^2 \text{ ie, } E \propto (\omega r)^2$$

or $(\omega_1 r_1)^2 = (\omega_2 r_2)^2$ or $\omega_1 r_1 = \omega_2 r_2$

or $10 \times 8 = \omega \times 5$ or $\omega = 16$ units

315 (c)

Velocity $v = \omega\sqrt{A^2 - x^2}$

and acceleration $= \omega^2 x$

Given, $\omega\sqrt{A^2 - x^2} = \omega^2 x$ or $\sqrt{A^2 - x^2} = \omega x$

Given, $T = \frac{2\pi}{\sqrt{3}}$

and $\omega = \frac{2\pi}{T} = \sqrt{3}$

substituting the value of ω in Eq. (i), we get

$$\sqrt{A^2 - x^2} = \sqrt{3}x$$

$\Rightarrow \quad A = 2x$

As amplitude $= \frac{\text{path length}}{2} \times 2$ cm

$\Rightarrow \quad x = 1$ cm

317 (a)

$$x_1 = A\sin(\omega t + \emptyset_1)$$
$$x_2 = A\sin(\omega t + \emptyset_2)$$
$$x_2 - x_1 = A[\sin(\omega t + \emptyset_2) - \sin(\omega t + \emptyset_1)]$$
$$= 2A\cos\left(\frac{2\omega t + \emptyset_1 + \emptyset_2}{2}\right)\sin\left(\frac{\emptyset_2 - \emptyset_1}{2}\right)$$

The resultant motion cam be treated as a simple harmonic motion with amplitude $2A$ $\sin\left(\frac{\emptyset_2 - \emptyset_1}{2}\right)$

Given, maximum distance between the particles $= x_0 + A$

$\therefore$ Amplitude of resultant SHM

$$= x_0 + A - x_0 = A$$
$$= 2A\sin\left(\frac{\emptyset_2 - \emptyset_1}{2}\right) = A$$
$$\emptyset_2 - \emptyset_1 = \frac{\pi}{3}$$

318 (a)

Time period of a simple pendulum of length l is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where g is acceleration due to gravity.

On moon

$$g_m = \frac{g}{6}$$

$$\therefore \quad T' = 2\pi\sqrt{\frac{l}{g/6}} = 2\pi\sqrt{\frac{6l}{g}} = \sqrt{6}. 2\pi\sqrt{\frac{l}{g}}$$

$\Rightarrow \quad T' = \sqrt{6}T$

Hence, time period increases on the surface of moon.

319 (b)

The motion of M is SHM, with length $CM = \sqrt{L^2 - d^2}$

$$\therefore \quad 2\pi\sqrt{\frac{(L^2 + d^2)^{1/2}}{g}}$$
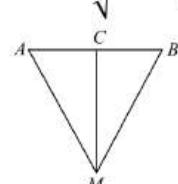
320 (d)

The simple pendulum at angular amplitude θ_0 is shown in the figure.

Maximum tension in the string is

$$T_{max} = mg + \frac{mv^2}{l}$$

...(i)

When bob of the pendulum comes from A to B, it covers a vertical distance h
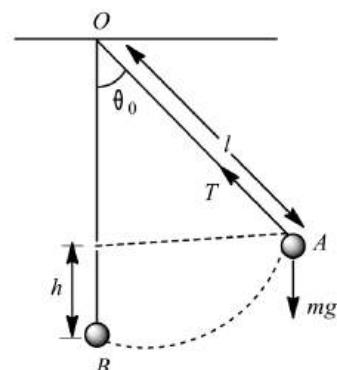
$$\therefore \quad \cos\theta_0 = \frac{l-h}{l}$$

$$\Rightarrow \quad h = l(1 - \cos\theta_0)$$

...(ii)

Also during A to B, potential energy of bob converts into kinetic energy ie, $mgh = \frac{1}{2}mv^2$

$$\therefore \quad v = \sqrt{2gh}$$

...(iii)

Thus, using Eqs. (i),(ii) and (iii), we obtain

$$T_{max} = mg + \frac{2mg}{l}l(1 - \cos\theta)$$
$$= mg + 2mg\left[1 - 1 + \frac{\theta_0^2}{2}\right]$$
$$= mg(1 + \theta_0^2)$$

321 (a)

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}};$$

$$n' = \frac{1}{2\pi}\sqrt{\frac{k'}{2m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{2m}} \quad [\because k' = 2k]$$

$$\therefore \quad n' = n$$

322 (b)

$$T_1 = 2\pi\sqrt{\frac{121}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{100}{g}}$$

So $T_1 > T_2$. Let the shorter pendulum makes n vibrations, then the longer pendulum will make one less than n vibrations to come in phase again

So $nT_2 = (n-1)T_1$

Or $n \times 2\pi\sqrt{\frac{100}{g}} = (n-1) \times 2\pi\sqrt{\frac{121}{g}}$

Or $10n = (n-1)11$ or $n = 11$

323 (b)

Here, $y = \sin^2 \omega t$
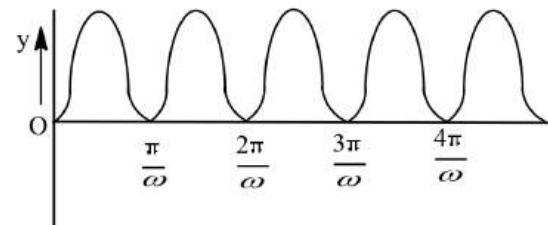
$$\frac{dy}{dt} = 2\omega \sin\omega t \cos\omega t$$

$$= \omega \sin 2\omega t$$

$$\frac{d^2 y}{dt^2} = 2\omega^2 \cos 2\omega t$$

For SHM, $\frac{d^2 y}{dt^2} \propto -y$

Hence, function is not SHM, but periodic. From the y-t graph, time period is

$$T = \frac{\pi}{\omega}$$

324 (b)

As periodic time is independent of amplitude

325 (d)

$$E = \frac{1}{2}m\omega^2 a^2 \Rightarrow E \propto a^2$$

326 (c)

The time period of the pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \qquad T \propto \sqrt{l}$$

Initially the centre of mass of the sphere is at the centre of the sphere. As the water slowly flows out of the hole at the bottom, the CM of the liquid (hollow sphere) first goes on downward and the upward. Hence, the effective length of the pendulum first increases and then decreases.

327 (b)

In accelerated frame of reference, a fictitious force (pseudo force) ma acts on the bob of pendulum as shown in figure
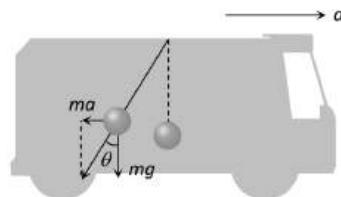
Hence,

$$\tan\theta = \frac{ma}{mg} = \frac{a}{g}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right) \text{ in the backward direction}$$

328 (b)

The motion of a load attached to a spring, when pulled and released, is a SHM. The motion of liquid contained in U-tube when it is compressed once in one limb and left to itself and the twisting motion in figure B are also simple harmonic.

329 (b)

$x = A\cos(\omega t + \theta)$;

Velocity, $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \theta)$

$$= -A\omega\sqrt{1 - \cos^2(\omega t + \theta)}$$

$$= -A\omega\sqrt{1 - x^2/A^2} = -\omega\sqrt{A^2 - x^2}$$

Here, $v = \pi \text{ cms}^{-1}$, $x = 1 \text{ cm}$, $\omega = \pi \text{s}^{-1}$

So $\pi = -\pi\sqrt{A^2 - 1^2}$

or $(-1)^2 = A^2 - 1$ or $A^2 = 2$

or $A = \sqrt{2}$ cm

330 (c)

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$$

331 (c)

$$y = a\sin\frac{2\pi}{T}t \Rightarrow \frac{a}{\sqrt{2}} = a\sin\frac{2\pi}{T}.t$$

$$\Rightarrow \sin\frac{2\pi}{T}t = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \Rightarrow \frac{2\pi}{T}t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8}$$

332 (b)

At B, the velocity is maximum. Using conservation of mechanical energy

$$\Delta PE = \Delta KE \Rightarrow mgH = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gH}$$

333 (a)

$U = k|x|^3 \Rightarrow F = -\frac{dU}{dx} = -3k|x|^2$ ...(i)

Also, for SHM $x = a \sin \omega t$ and $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\Rightarrow$ acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow F = ma$

$= m\frac{d^2x}{dt^2} = -m\omega^2 x$ ...(ii)

From equation (i) & (ii) we get $\omega = \sqrt{\frac{3kx}{m}}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(a \sin \omega t)}} \Rightarrow T$

$$\propto \frac{1}{\sqrt{a}}$$

334 (b)

$\frac{v_{\max}}{A_{\text{mas}}} = \frac{a\omega}{a\omega^2} = \frac{1}{\omega}$

335 (a)

In SHM for a complete cycle average value of kinetic energy and potential energy are equal, $i.e.,$

$< E > = < U > = \frac{1}{4}m\omega^2 a^2$

336 (a)

If v is velocity of pendulum at Q
and 10% energy is lost while moving from P to Q
Hence, by applying conservation of energy
between P and Q

$\frac{1}{2}mv^2 = 0.9\,(mgh) \Rightarrow v^2 = 2 \times 0.9 \times 10 \times 2 \Rightarrow v$

$= 6m/s$

337 (c)

Acceleration $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{0.5}{0.02}} = 5$

Maximum velocity $v_{\max} = a\omega = 0.1 \times 5 = 0.5$

338 (b)

$Mg = Kl \Rightarrow U_{\max} = \frac{1}{2}K(2l)^2 = 2Kl$

339 (a)

As a is the side of cube σ is its density
Mass of cube $a^3\sigma$, its weight $= a^3\sigma g$
Let h be the height of cube immersed in liquid of density of ρ in equilibrium then, $F = a^2 h\rho g = Mg = a^3\sigma g$
If it is pushed down by y then the buoyant force
$F' = a^2(h+y)\rho g$
Restoring force is $\Delta F = F' - F = a^2(h+y)\rho g - a^2 h\rho g$
$= a^2 y \rho g$

Restoring acceleration $= \frac{\Delta F}{M} = -\frac{a^2 y\rho g}{M} = -\frac{a^2\rho g}{a^3\rho}y$

Motion is S.H.M.

$\Rightarrow T = 2\pi\sqrt{\frac{a^3\sigma}{a^2\rho g}} = 2\pi\sqrt{\frac{a\sigma}{\rho g}}$

340 (b)

Let k be the force constant of spring of length l_2.
Since $l_1 = n\,l_2$, where n is an integer, so the spring is made of $(n+1)$ equal parts in length each of length l_2

$\therefore \frac{1}{k} = \frac{(n+1)}{k}$ or $k = (n+1)k$

The spring of length $l_1 (= n\,l_2)$ will be equivalent to n springs connected in series where spring

constant $k' = \frac{k}{n} = \frac{(n+1)k}{n}$

341 (a)

Simple harmonic waves are set up in a string fixed at the two ends

342 (c)

$2 = 2\pi\sqrt{\frac{m}{k}}$; and $3 = 2\pi\sqrt{\frac{m+2}{k}}$

So, $\frac{3}{2} = \sqrt{\frac{m+2}{k}}$

or $9m = 4m + 8$ or $m = 1.6$ kg

343 (d)

$TE = \frac{1}{2}m\omega^2 a^2$

$TE \propto a^2$

344 (d)

Let l be the length of block immersed in liquid as shown in the figure. When the block is floating.
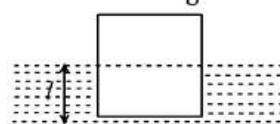
$\therefore mg = Al\rho g$

If the block is given vertical displacement y then the effective restoring force is

$F = -[A(l+y)\rho g - mg)$
$\qquad = -[A(l+y)\rho g - Al\rho g]$
$\qquad = -(Al\rho g)y$

$i.e., F \propto y$. As this F is directed towards its equilibrium position of block, so if the block is left free, it will execute simple harmonic motion.
Here inertia factor = mass of block $= m$
Spring factor $= A\rho g$

$\therefore$ Time period $= T = 2\pi\sqrt{\frac{m}{A\rho g}}$

$i.e. T \propto \frac{1}{\sqrt{A}}$

345 (d)

We have
$$A = A_0 e^{bt/2m}$$
In this case after 6 s amplitude becomes $\frac{1}{27}$ times.

346 (b)
Motion given here is SHM starting from rest

347 (d)
At the mean position, the speed will be maximum
$$v_{max} = 16 \text{ cms}^{-1} = a\omega = 4\omega,$$
So, $\omega = \frac{16}{4} = 4$ rad/s
and $v = \omega\sqrt{A^2 - y^2}$
$$8\sqrt{3} = 4\sqrt{A^2 - y^2}$$
or $A^2 - y^2 = 12$
or $y^2 = A^2 - 12 = (4)^2 - 12 = 4$
or $y = 2$ cm

348 (a)
The given equation is written as,
$$y = 3\sin\left(100\,t + \frac{\pi}{6}\right)$$
...(i)
The general equation of simple harmonic motion is written as
$$y = a\sin(\omega t + \emptyset)$$
...(ii)
Equating Eqs. (i) and (ii), we get
$$a = 3, \omega = 100$$
Maximum velocity, $v = a\omega$
$$= 3 \times 100 = 300 \text{ ms}^{-1}$$

349 (a)
In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions.
Therefore, the time taken for the particle to go from 0 to $A/2$ will be less than the time taken to go it from $A/2$ to A.
Hence, $T_1 < T_2$

351 (d)
$$|A| = \omega^2 x \Rightarrow \frac{|A|}{x} = \omega^2$$
From the given value $\frac{|A|}{x} = \omega^2 = 4 \Rightarrow \omega = 2$
Also $\omega = \frac{2\pi}{T} \Rightarrow 2 = \frac{2\pi}{T} \Rightarrow T = \pi \sec$

352 (d)
Time period, $T = 2\pi\sqrt{\frac{L}{g}}$

And $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$
Displacement, $x = a\cos\omega t = a\cos\sqrt{\frac{g}{L}}t$

353 (d)
$$x = a\sin^2\omega t = \frac{a}{2}(1 - \cos 2\omega t)$$

354 (b)
By cutting spring in four equal parts force constant (K) of each parts becomes four times $\left(\because k \propto \frac{1}{l}\right)$ so by using $T = 2\pi\sqrt{\frac{m}{K}}$; time period will be half $i.e. T' = T/2$

355 (b)
For forced oscillation,
$x = x_0\sin(\omega t + \phi)$ and $F = F_0\cos\omega t$
where, $x_0 = \frac{F_o}{m(\omega_o^2 - \omega^2)} \propto \frac{1}{m(\omega_o^2 - \omega^2)}$

356 (b)
Let T_1 and t_2 be the time periods of the pendulums with lengths 1.0 m and 1.21 m respectively
$$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{1.21}{1}} = 1.1 \quad ...(i)$$
Let v_1 and v_2 be the vibrations made by two pendulum to swing together
$\therefore v_1 T_1 = v_2 T_2$ ...(ii)
For the two pendulums to swing together, required condition is
$v_1 - v_2 = 1$
or $v_1 = v_2 + 1$
$\therefore (v_2 + 1)T_1 = v_2 T_2$
or $(v_2 + 1)v_2 = T_2/T_1 = 1.1$
or $1 + \frac{1}{v_2} = 1.1$
or $\frac{1}{v_2} = 1.1 - 0.1 = \frac{1}{10}$
or $v_2 = 10$

357 (a)
Velocity $v = a\omega = a \times 2\pi n$
$= 0.06 \times 2\pi \times 15 = 5.65$ m/s
Acceleration $A = \omega^2 a = 4\pi^2 n^2 a = 5.32 \times 10^2 m/s^2$

358 (c)
$AC = l\cos\theta$
$\therefore OC = OA - AC$
$= l - l\cos\theta = l(1 - \cos\theta)$
Max. KE of bob at O =Max. PE of bob at B
$= mg \times OC = mgl(1 - \cos\theta)$

359 (d)

This is the case of freely falling lift and in free fall of lift effective g for pendulum will be zero. So

$$T = 2\pi \sqrt{\frac{l}{0}} = \infty$$

360 (a)

Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decreases in volume, then considering the process to take place gradually (i.e. isothermal)
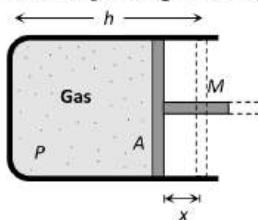
$P_1 V_1 = P_2 V_2$
$\Rightarrow PV = (P + \Delta P)(V - \Delta V)$
$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$
$\Rightarrow \Delta P.V - P.\Delta V = 0$ (neglecting $\Delta P.\Delta V$)

$$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.x}{h}$$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M

Hence $F = \Delta P.A = \frac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \frac{PA}{h}$

$$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi \sqrt{\frac{Mh}{PA}}$$

Short trick: by checking the options dimensionally. Option (a) is correct

361 (b)

Total energy, $E = \frac{1}{2}m\omega^2 a^2$;

KE$= \frac{3E}{4} - \frac{1}{2}m\omega^2(a^2 - y^2)$

So $\frac{3}{4} = \frac{a^2 - y^2}{a^2}$ or $y^2 = a^2/4$ or $y = a/2$

362 (c)

Maximum potential energy position $y = \pm a$ and maximum kinetic energy position is $y = 0$

363 (c)

Total energy in SHM $E = \frac{1}{2}m\omega^2 a^2$; (where $a =$ amplitude)

Kinetic energy $K = \frac{1}{2}m\omega^2(a^2 - y^2) = E - \frac{1}{2}m\omega^2 y^2$

When $y = \frac{a}{2} \Rightarrow K = E - \frac{1}{2}m\omega^2\left(\frac{a^2}{4}\right) = E - \frac{E}{4} = \frac{3E}{4}$

364 (b)

Kinetic energy, $E_k = \frac{1}{2}m(a^2 - y^2)$

$$= \frac{1}{2}m\left(a^2 - \frac{a^2}{4}\right) = \frac{1}{2}m\omega^2 a^2 \frac{3}{4}$$

Total energy, $E = \frac{1}{2}m\omega^2 a^2$

So $E_k/E = 3/4$

365 (d)

Time period of simple pendulum in air

$$T = 2\pi \sqrt{\frac{l}{g}}$$
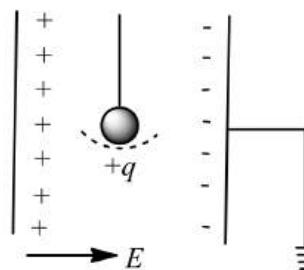
When it is suspended between vertical plates of a charged parallel plate capacitor, then acceleration due to electric field,

$$a = \frac{qE}{m}$$

This acceleration is acting horizontally and acceleration due to gravity is acting vertically. So, effective acceleration

$$g' = \sqrt{g^2 + a^2} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

Hence, $T' = 2\pi \sqrt{\dfrac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$

366 (b)

Energy of oscillation, $E = \alpha A^4$
Kinetic energy of mass at $x = x$ is
$K = E - U = \alpha(A^4 - x^4)$
As $K = 3U$
$\alpha(A^4 - x^4) = 3\alpha x^4$ or $x = \pm \frac{A}{\sqrt{2}}$

368 (a)

$F = -kx \Rightarrow dW - Fdx = -kxdx$
So $\int_0^W dW = \int_0^x -kx\,dx \Rightarrow W = U = \frac{1}{2}kx^2$

369 (b)

Time period $T = 2\pi \sqrt{\frac{m}{k}}$
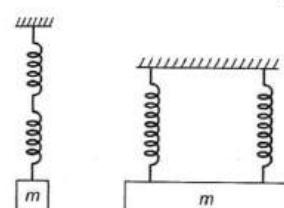
$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}}$

$$\frac{2}{T} = \sqrt{\frac{2k}{\frac{k}{2}}} = 2$$
$$T = 1 \text{ s}$$

370 (a)

Tension in the spring when bob passes through lowest point

$$T = mg + \frac{mv^2}{r} = mg + mv\omega \quad [\because v = r\omega]$$

putting v $= \sqrt{2gh}$ and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

we get $T = m(g + \pi\sqrt{2gh})$

371 (d)

Given spring system has parallel combination, so

$K_{eq} = K_1 + K_2$ and time period $T = 2\pi\sqrt{\frac{m}{(K_1+K_2)}}$

372 (a)

Time period $T \propto \sqrt{l} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

Also according to thermal expansion $l' = l(1 + \alpha\Delta\theta)$

$$\frac{\Delta l}{l} = \alpha\Delta\theta$$

Hence $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times (40 - 20) = 12 \times 10^{-5}$$

$\Rightarrow \Delta T = 12 \times 10^{-5} \times 86400 \ seconds/day$

$\therefore \Delta T \approx 10.3 \ seconds/day$

373 (c)

$x = 0.01 \sin 100\pi \, (t + 0.05)$

$= 0.01 \sin(100\pi t + 5\pi)$

$\therefore$ Angular frequency $\omega = 100\pi = \frac{2\pi}{T}$

or $T = \frac{2}{100} = 0.02$s

374 (c)
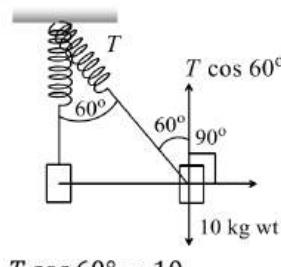
$T\cos 60° = 10$

$$T = \frac{10}{\cos 60°} = 20 \ kgwt$$

375 (d)

$y = a \sin\omega t + b \cos\omega t$

Let $a = r\cos\theta$ and $b = r\sin\theta$

$y = r\cos\theta\sin\omega t + r\sin\theta\cos\omega t$

$= r\sin(\omega t + \theta)$

$\therefore a^2 + b^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2$

or $r = \sqrt{a^2 + b^2}$

376 (c)

$v = \omega\sqrt{a^2 - y^2} \Rightarrow 10 = \omega\sqrt{a^2 - (4)^2}$ and $8 = \omega\sqrt{a^2 - (5)^2}$

On solving $\omega = 2 \Rightarrow \omega = \frac{2\pi}{T} = 2 \Rightarrow T = \pi s$

377 (b)

As we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_{earth}}{g_{planet}} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$\Rightarrow T_p = 2\sqrt{2} \ s$

378 (b)

$$\omega = \frac{2\pi}{T} = 100\pi \Rightarrow T = 0.02 \ s$$

379 (c)

$$T_1 = 2\pi\sqrt{\frac{M}{k_1}} \quad \text{or} \quad k_1 = \frac{4\pi^2 M}{T_1^2}$$

And $k_2 = \frac{4\pi^2 M}{T_2^2}$

In series combination,

$$k_{eff} = \frac{k_1 k_2}{k_1 + k_2} = \frac{4\pi^2 M}{T_1^2 + T_2^2}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{M}{k_{eff}}} = \sqrt{T_1^2 + T_2^2}$$

380 (a)

$$v_{max} = a\omega = a \times \frac{2\pi}{T} \Rightarrow a = \frac{v_{max} \times T}{2\pi}$$

$$A = \frac{1.00 \times 10^3 \times (1 \times 10^{-5})}{2\pi} = 1.59 \ mm$$

381 (a)

Potential energy is minimum (in this case zero) at mean position ($x = 0$) and minimum at extreme position ($x = \pm A$)

At time $t = 0, x = A$, hence potential should be maximum. Therefore graph. I is correct. Further in graph III. Potential energy is minimum at $x = 0$, hence this is also correct.

382 (a)

The standard differential equation in satisfied by only the function $\sin\omega t - \cos\omega t$. Hence it represents S.H.M.

383 (d)

In damped oscillation, amplitude goes on decaying exponentially

$a = a_0 e^{-bt}$ where b = damping coefficient

Initially, $\frac{a_0}{3} = a_0 e^{-b \times 100T}$, $T = $ time of one oscillation

or $\frac{1}{3} = e^{-100bT}$ ...(i)

Finally $a = a_0 e^{-b \times 200T}$ or $a = a_0[e^{-100bT}]^2$

or $a = a_0 \times \left[\frac{1}{3}\right]^2$ [From (i)]

or $a = a_0/9$

384 (c)

$$n = \frac{1}{2\pi}\sqrt{\frac{K_{effective}}{m}} = \frac{1}{2\pi}\sqrt{\frac{(K+2K)}{m}} = \frac{1}{2\pi}\sqrt{\frac{3K}{m}}$$

385 (d)

The equation of SHM is

$\quad\quad y = A\sin(\omega t + \emptyset)$

or $\quad y = A(\sin\omega t \cos\emptyset + \cos\omega t \sin\emptyset)$

...(i)

The given expression is

$\quad\quad y = 2(\sin 5\pi t + \sqrt{2}\cos\pi t)$

...(ii)

$A\cos\emptyset = 2$ and $A\sin\emptyset = 2\sqrt{2}$

Squaring and adding, we get

$\quad\quad A = 2\sqrt{3}$

386 (d)

$\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}\frac{rad}{sec}$ (For $y = 2\ cm$) 2

$\quad\quad = 4\left(\sin\frac{\pi}{6}t_1\right)$

By solving $t_1 = 1 sec$ (For $y = 4\ cm$) $t_2 = 3\ sec$

So time taken by particle in going from $2\ cm$ to extreme position is $t_2 - t_1 = 2\ sec$. Hence required ratio will be $\frac{1}{2}$

387 (d)

$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; If $m = k$, then $v = \frac{1}{2\pi}$

388 (a)

Given,

$\quad\quad y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$

$\therefore \quad \frac{dy_1}{dt} = v_1 = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

or $\quad v_1 = 10\pi\sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

or $\quad v_1 = 10\pi\sin\left(100\pi t + \frac{5\pi}{6}\right)$

and $\quad y_2 = 0.1\cos\pi t$

$\therefore \quad \frac{dy_2}{dt} = v_2 = -0.1\sin\pi t = 0.1\sin(\pi t + \pi)$

Hence, phase difference

$\quad\quad \Delta\emptyset = \emptyset_1 - \emptyset_2 = \left(100\pi t + \frac{5\pi}{6}\right) - (\pi t + \pi)$

$\quad\quad = \frac{5\pi}{6} - \pi$ (at $t = 0$)

$\quad\quad = -\frac{\pi}{6}$

389 (a)

PE of body in SHM at an instant,

$$U = \frac{1}{2}k(a - x)^2 = \frac{1}{2}k(x - a)^2$$

If the displacement, $y = (a - x)$ then

$$U = \frac{1}{2}k(a - x)^2 = \frac{1}{2}k(x - a)^2$$

390 (b)

$T = 2\pi\sqrt{\frac{m}{K}}$. Also spring constant $(K) \propto \frac{1}{\text{Length }(l)}$,

when the spring is half in length, then K becomes twice

$\therefore T' = 2\pi\sqrt{\frac{m}{2K}} \Rightarrow \frac{T'}{T} = \frac{1}{\sqrt{2}} \Rightarrow T' = \frac{T}{\sqrt{2}}$

391 (a)

Kinetic energy varies with time but is never negative

392 (c)

PE$= \frac{1}{2}m\omega^2 y^2 = \frac{1}{4} \times \frac{1}{2}m\omega^2 a^2$

or $y^2 = \frac{a^2}{4}$

or $y = \pm\frac{a}{2}$

393 (d)

If a force acting on an object is a function of position only, it is said to be conservative force and it can be represented by a potential energy (U) function which for one dimensional case satisfies the derivative condition.

$$F(x) = -\frac{dU}{dx}$$

Given, $\quad\quad U_x = k[1 - \exp(-x^2)]$

$\therefore \quad\quad F = -\frac{dU}{dx} = -2kx\exp(-x^2)$

At $x = 0, F = 0$.

Hence, at equilibrium force exerted on particle is zero.

Also, potential energy of the particle is minimum at $x = 0$ and $x = \pm\infty$ the potential energy is maximum. Hence, at $x = 0$, the motion of particle is simple harmonic.

394 (c)

Given elastic energies are equal $i.e., \frac{1}{2}k_1 x_1^2 = \frac{1}{2}k_2 x_2^2$

$$\Rightarrow \frac{k_1}{k_2} = \left(\frac{x_2}{x_1}\right)^2 \text{ and using } F = kx$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{k_1 x_1}{k_2 x_2} = \frac{k_1}{k_2} \times \sqrt{\frac{k_2}{k_1}} = \sqrt{\frac{k_1}{k_2}}$$

395 (a)

$$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow m \propto T^2 \Rightarrow \frac{m_2}{m_1} = \frac{T_2^2}{T_1^2}$$

$$\Rightarrow \frac{M+m}{M} = \left(\frac{\frac{5}{4}T}{T}\right)^2 \Rightarrow \frac{m}{M} = \frac{9}{16}$$

396 (d)

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

$$\text{and} \quad f' = \frac{1}{2\pi}.2\sqrt{\frac{k_1+k_2}{m}} = 2f$$

397 (a)

Velocity of a particle executing S.H.M. is given by

$$v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{2\pi}{T}\sqrt{\frac{3A^2}{4}}$$

$$= \frac{\pi A\sqrt{3}}{T}$$

398 (c)

Energy stored= work done

So $E = \frac{1}{2}kr^2$

or $r = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\times 2}{400}} = \frac{1}{10}$ m

$$a = \omega^2 r = \left(\sqrt{\frac{k}{m}}\right)^2 \times \frac{1}{10}$$

$$= \left(\frac{400}{1}\right) \times \frac{1}{10} = 40 \text{ ms}^{-2}$$

399 (c)

If $y_1 = a_1\sin\omega t$ and $y_2 = a_2\sin(\omega t + 0) = a_2\sin\omega t$

$$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} - \frac{2y_1 y_2}{a_1 a_2} = 0 \Rightarrow y_2 = \frac{a_2}{a_1}y_1$$

This is the equation of straight line

400 (c)

Time period of a simple pendulum of length l, is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

...(i)

Where, g is acceleration due to gravity.

When $g' = \frac{g}{4}$,

New, time period is

$$T' = 2\pi\sqrt{\frac{l}{g/4}}$$

...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{T'}{T} = \sqrt{\frac{g}{g/4}} = 2$$

$$\Rightarrow \qquad T' = 2T$$

Hence, new time period becomes twice of the original value.

401 (c)

PE, $V = \frac{1}{2}m\omega^2 x^2$

and KE, $T = \frac{1}{2}m\omega^2(a^2 - x^2)$

$$\therefore \frac{T}{V} = \frac{a^2 - x^2}{x^2}$$

403 (c)

In case of SHM, when motion is considered from the equilibrium position, velocity at an instant t is given by

$$v = \omega\sqrt{A^2 - y^2}$$

At the mean or equilibrium position ie, when $y = 0$

$$v = v_{\max} = \omega A$$

At the extreme positions, ie, when $y = \pm A$

$$v = v_{\min} = 0$$

Hence, velocity is maximum at equilibrium position.

404 (a)

As restoring force $F \propto -(y)$, so graph (c) represents the correct relation between F and t.

405 (d)

$v_{\max} = A\omega$

When A becomes twice $v_{\max}$ is also doubled.

406 (c)

$$K \propto \frac{1}{l} \Rightarrow Kl = K' \times \frac{l}{2} \Rightarrow K' = 2K$$

407 (a)

Electric intensity at B due to sheet of charge,

$$E = \frac{1}{2}\frac{\sigma}{\varepsilon_0}$$

force on the bob due to sheet of charge

$$F = qE = \frac{1}{2}\frac{\sigma q}{\varepsilon_0}$$

As the bob is in equilibrium, so
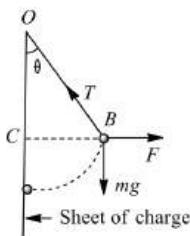
$$\frac{mg}{OC} = \frac{F}{CB} = \frac{T}{BO}$$

$$\tan\theta = \frac{CB}{OC} = \frac{F}{mg}$$

$$= \frac{\frac{1}{2}\sigma q/\varepsilon_0}{mg} = \frac{\sigma q}{2\varepsilon_0 mg}$$

408 (d)

For damped motion $a = a_0 e^{-bt}$

For first case,

$$\frac{a_0}{3} = a_0 e^{-b\times 2} \ or \ \frac{1}{3} = e^{-2b}$$

For second case,

$$\frac{a_0}{n} = a_0 e^{-b\times 6} \ or \ \frac{1}{n} = e^{-6b} = \left(e^{-2b}\right)^3 = \left(\frac{1}{3}\right)^3 \ n$$
$$= 3^3$$

409 (d)

$$A_{max} = a\omega^2 \Rightarrow a = \frac{A_{max}}{\omega^2} = \frac{7.5}{(3.5)^2} = 0.61 \ m$$

410 (d)

The equation is given

$$y = 5\sin\pi(t+4) \qquad ...(i)$$

The standard equation is

$$y = A\sin(\omega t + \emptyset) \qquad ...(ii)$$

Now comparing Eqs. (i) and (ii), we get

$$A = 5\ \text{m}, \ \omega t = \pi t \quad or \quad \omega = \pi$$

So, $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\ \text{s}$

411 (b)

$$\frac{\text{Potential energy } (U)}{\text{Total energy } (E)} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2}$$

So $\frac{2.5}{E} = \frac{\left(\frac{a}{2}\right)^2}{a^2} \Rightarrow E = 10J$

412 (b)

We both the masses are there, then angular frequency $\omega = \sqrt{k/(m_1 + m_1)}$; is there, then $\omega' = \sqrt{k/m_2}$

413 (a)

At mean position, the kinetic energy is maximum

Hence $\frac{1}{2}ma^2\omega^2 = 16$

On putting the values we get

$$\omega = 10 \Rightarrow T = \frac{2\pi}{\omega} = \frac{\pi}{5}s$$

414 (d)

Maximum KE$= \frac{1}{2}m\omega^2 A^2$; minimum KE=0

Average KE$= \frac{0+\frac{1}{2}m\omega^2 A^2}{2} = \frac{1}{4}m\omega^2 A^2$

Similarly average PE$= \left(\frac{0+\frac{1}{2}m\omega^2 A^2}{2}\right)/2$

$$= \frac{1}{4}m\omega^2 A^2$$

415 (d)

From the given equation, $a = 5$ and $\omega = 4$

$\therefore v = \omega\sqrt{a^2 - y^2} = 4\sqrt{(5)^2 - (3)^2} = 16$

416 (a)

Velocity, $v = r\omega\cos\omega t$;

$$0.4 = r \times \frac{2\pi}{16}\cos\frac{2\pi}{16}\times 2 = r\times\frac{2\pi}{16}\times\frac{1}{\sqrt{2}}$$

or $\ r = \frac{0.4\times 16\times\sqrt{2}}{2\pi} = \frac{3.2\sqrt{2}}{\pi} = 1.44$ m

417 (c)

Maximum velocity $V_{max} = a\omega$

$$\omega = \frac{2\pi}{T} \ \therefore V_{max} = \frac{2\pi a}{T}, i.e., V \propto \frac{a}{T}$$

$$\therefore \frac{V_1}{V_2} = \frac{a_1}{a_2}\times\frac{T_2}{T_1} = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$$

$$\therefore V_2 = 6V_1 = 6V$$

418 (c)

$y = a\sin(2\pi nt + \alpha)$. Its phase at time $t = 2\pi nt + \alpha$

419 (b)

Angular velocity $\omega = \sqrt{\left(\frac{k}{m}\right)} = \sqrt{\left(\frac{10}{10}\right)} = 1$

Now $v = \omega\sqrt{a^2 - y^2} \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2} = (0.5)^2 - \frac{(0.4)^2}{1^2}$

$\Rightarrow y^2 = 0.9 = y = 0.3 \ m$

420 (c)

$d^2x/dt^2 = -kx$;

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

So $\ T = 2\pi\sqrt{\frac{x}{kx}} = 2\pi\sqrt{\frac{1}{k}}$

421 (a)

The Lissajous figure will be parabola if period ratio is $1:2$ and phase difference is $\pi/2$

Let $x = a\sin(2\omega t + \pi/2)$ and $y = b\sin\omega t$

$\therefore \ \sin\omega t = y/b$

Now, $\frac{x}{a} = \sin(2\omega t + \pi/2) = \cos 2\omega/t$

$$= 1 - 2\sin^2 \omega t = 1 - \frac{2y^2}{b^2}$$

or $\frac{2y^2}{b^2} = 1 - \frac{x}{a} = -\left(\frac{x-a}{a}\right)$

or $y^2 = -\frac{b^2}{2a}(x - a)$

it is an equation of a parabola as given in figure

423 (a)

$x = 3\sin 2t + 4\cos 2t$. From given equation

$a_1 = 3, a_2 = 4$ and $\phi = \frac{\pi}{2}$

$\therefore a = \sqrt{a_1^2 + a_2^2} = \sqrt{3^2 + 4^2} = 5 \Rightarrow v_{max} = a\omega$

$$= 5 \times 2 = 10$$

424 (b)

If two SHMs act in perpendicular directions, then their resultant motion is in the form of a straight line or a circle or a parabola etc, depending on the frequency ratio of two SHMs and initial phase difference.

For the frequency ratio $\omega_1 : \omega_2 = 2 : 1$, the two perpendicular SHMs are

$$x = a\sin(\omega_1 + \emptyset) \text{ and } y = b\sin\omega_2 t$$

For figure of eight $\emptyset = 0, \pi, 2\pi$

426 (b)

As it is clear that in vacuum, the bob will not experience any frictional force. Hence, there shall be no dissipation therefore, it will oscillate with constant amplitude

427 (b)

$$\frac{U}{E} = \frac{\frac{1}{2}m\omega^2 y^2}{\frac{1}{2}m\omega^2 a^2} = \frac{y^2}{a^2} = \frac{\left(\frac{a}{2}\right)^2}{a} = \frac{1}{4}$$

428 (a)

$x = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$

$= 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\cos 2\pi t$

$x = 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\sin\left(2\pi t + \frac{\pi}{2}\right)$

Phase difference between constituent waves $\phi = \frac{\pi}{2}$

$\therefore$ Resultant amplitude $A = \sqrt{\left(5\sqrt{2}\right)^2 + \left(5\sqrt{2}\right)^2} =$

$10\ cm$

429 (c)

At equilibrium position, potential energy of the body is zero. So, the total energy at equilibrium position is completely kinetic energy.

430 (b)

$y_1 = 5[\sin 2\pi t + \sqrt{3}\cos 2\pi t]$

$= 10[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t]$

$= 10[\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t]$

$= 10[(\sin 2\pi t + \frac{\pi}{3})]$

$\Rightarrow A_1 = 10$

Similarly, $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

$\Rightarrow A_2 = 5$

Hence, $\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

431 (a)

The given equation is

$$y = 5\sin\frac{\pi}{2}(100t - x)$$

...(i)

Comparing Eq. (i) with standard wave equation, given by

$$y = A\sin(\omega t - kx)$$

...(ii)

we have

$$\omega = \frac{100\pi}{2} = 50\pi$$

$\Rightarrow \frac{2\pi}{T} = 50\pi$

$\Rightarrow T = \frac{2\pi}{50\pi} = 0.04$ s

432 (d)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

Given, $\frac{\Delta l}{l} = 21\%$

$\therefore \frac{\Delta T}{T} = \frac{1}{2} \times 21\% = 10.5\%$

433 (b)

$$E_1 = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2E_1}{K}}$$

$$E_2 = \frac{1}{2}Ky^2 \Rightarrow y = \sqrt{\frac{2E_2}{K}}$$

and $E = \frac{1}{2}K(x + y)^2 \Rightarrow x + y = \sqrt{\frac{2E}{K}}$

$\Rightarrow \sqrt{\frac{2E_1}{K}} + \sqrt{\frac{2E_2}{K}} = \sqrt{\frac{2E}{K}} \Rightarrow \sqrt{E_1} + \sqrt{E_2} = \sqrt{E}$

434 (b)

$$p_{max} = \sqrt{2m\ E_{max}}$$

435 (a)

Average kinetic energy of particle

$$= \frac{1}{4} ma^2 \omega^2$$

$$= \frac{1}{4} ma^2 (2\pi v)^2 = \pi^2 v^2 ma^2$$

436 (b)

The amplitude of oscillations will be the maximum compression is the spring. At the time of maximum compression velocities of both the blocks are equal say v, then using law of conservation of momentum,

$m_1 v_0 = (m_1 + m_2)v$

or $1 \times 12 = (1 + 2)v$ or $v = 4 \, \text{cms}^{-1}$

Using law of conservation of energy, we have

$$\frac{1}{2} m_1 v_0^2 = \frac{1}{2} kx^2 + \frac{1}{2} (m_1 + m_2)v^2$$

Putting the value and solving we get $x = 2$ cm

437 (b)

Amplitude of damped oscillator

$A = A_0 e^{-\lambda t}; \lambda = $ constant, $t = $ time

For $t = 1$ min. $\frac{A_0}{2} = A_0 e^{-\lambda t} \Rightarrow e^\lambda = 2$

For $t = 3$ min. $A = A_0 e^{-\lambda \times 3} = \frac{A_0}{(e^\lambda)^3} = \frac{A_0}{2^3} \Rightarrow X = 2^3$

438 (b)

According to the law of conservation of mechanical energy, we get
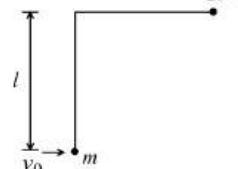
$$\frac{1}{2} mv_0^2 = mgl$$

$$\Rightarrow v_0 = \sqrt{2gl}$$

439 (d)

It is the least interval of time after which the periodic motion of a body repeats itself. Therefore, displacement will be zero.

440 (c)

$$T^2 = 4\pi^2 \left(\frac{l}{g}\right)$$

...(i)

$$T_1^2 = 4\pi^2 \left(\frac{l+10}{g}\right)$$

...(ii)

$$T_2^2 = 4\pi^2 \left(\frac{l-10}{g}\right)$$

... (iii)

Adding Eqs. (ii) and (iii), we get

$$T_1^2 + T_2^2 = 4\pi^2 \left[\frac{2l}{g}\right]$$

$$= 2(4\pi^2)\left(\frac{l}{g}\right) = 2T^2$$

441 (a)

As we go from equator to pole the value of g increases. Therefore time period of simple pendulum decreases

$$\left(\because T \propto \frac{1}{\sqrt{g}}\right)$$

442 (a)

Spring constant of each part, $k' = 2k$

Original frequency of system

$$\alpha = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

where m is the mass of the block.

New frequency of system

$$\alpha' = \frac{1}{2\pi} \sqrt{\frac{k'}{m}}$$

or $$\alpha' = \frac{1}{2\pi} \sqrt{\frac{2k}{m}}$$

$\therefore$ $$\alpha' = \sqrt{2}\alpha$$

443 (d)

In equilibrium, $T = mg$

Work done$= mg = mgx = \frac{1}{2} kx^2$

or $x = \frac{2mg}{k} = \frac{2T}{k}$

Energy stored$= mgx = Tx$

$$= T \times \frac{2T}{k} = \frac{2T^2}{k}$$

444 (c)

Displacement $x(t) = 5 \cos\left(2\pi \times \frac{3}{2} + \frac{\pi}{4}\right)$

$$= 5 \cos\left(\frac{13\pi}{4}\right) = -3.5 \text{ m}$$

$$y = 5 \cos\left(2\pi t + \frac{\pi}{4}\right)$$

$\therefore$Velocity $(v) = \frac{dy}{dt} = -10\pi \sin\left(2\pi t + \frac{\pi}{4}\right)$

$\therefore$Acceleration $a = \frac{dv}{dt} = -20\pi^2 \cos\left(2\pi t + \frac{\pi}{4}\right)$

$$= -20\pi^2 \cos\left(2\pi t \frac{3}{2} + \frac{\pi}{4}\right)$$

$$= 20\pi^2 \cos\frac{13\pi}{4} = 140 \text{ m/s}^2$$

445 (c)

$$n = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \Rightarrow \frac{n_S}{n_P} = \sqrt{\frac{K_S}{K_P}} \Rightarrow \frac{n_S}{n_p} = \sqrt{\frac{\left(\frac{K}{2}\right)}{2K}} = \frac{1}{2}$$

446 (a)

The frequency will be same $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ but due to the constant qE force, the equilibrium position

gets shifted by $\frac{qE}{K}$ in forward direction. So Sol. will be (a)

447 (c)

$$T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2\ s$$

448 (b)

When mass 700 g is removed, the left out mass $(500 + 400)g$ oscillates with a period of 3 s

$$\therefore 3 = t = 2\pi\sqrt{\frac{(500+400)}{k}} \quad ...(i)$$

When 500 g mass is also removed, the left out mass is 400 g

$$\therefore t' = 2\pi\sqrt{\frac{400}{k}} \quad ...(ii)$$

$$\Rightarrow \frac{3}{t'} = \sqrt{\frac{900}{400}} \Rightarrow t' = 2s$$

449 (b)

$F = mg = kx$;

For first case,

$$k = \frac{mg}{x} = \frac{1 \times 10N}{0.05\ m} = 200\ Nm^{-1}$$

For second case,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2.0}} = \sqrt{100} = 10Hz$$

$$r = \frac{m'g}{k} = \frac{2 \times 10}{200} = 0.1m$$

$$\therefore v_{max} = r\omega = 0.1 \times 10 = 1\ ms^{-1}$$

451 (d)

Here the restoring force of charge Q is inversely proportional to the square of the distance, hence the motion will be oscillatory but not SHM, for which restoring force $\propto$ displacement

452 (d)

Effective value of $'g'$ remains unchanged

453 (c)

Potential energy $= \frac{1}{2}m\omega^2y^2 = \frac{1}{4} \times \frac{1}{2}m\omega^2a^2$

$$y^2 = \frac{a^2}{4} \Rightarrow y = \pm\frac{a}{2}$$

454 (b)

Standard equation for given condition

$$x = a\cos\frac{2\pi}{T}t \Rightarrow x = -0.16\cos(\pi t)$$

[As $a = -0.16\ meter, T = 2\ s$]

455 (d)

$A = -\omega^2y$ at mean position $y = 0$

So acceleration is minimum (zero)

456 (c)

$x = 3\sin 20\pi t + 4\cos 20\pi t$

$$= 5\left[\frac{3}{5}\sin 20\pi t + \frac{4}{5}\cos 20\pi t\right]$$

$$= 5[\cos\theta\sin 20\pi t + \sin\theta\cos 20\pi t]$$

$$= 5\sin(20\pi t + \theta)$$

It is a SHM of amplitude 5 cm.

457 (d)

$$v_{max} = a\omega = \frac{a.2\pi}{T} = \frac{2\pi a}{T}$$

458 (b)

Maximum acceleration = Maximum velocity $\times$ π

ie, $\quad \omega^2A = \pi\omega A$

where A is amplitude and ω is angular velocity.

$$\Rightarrow \quad \omega = \pi$$

$$\Rightarrow \quad \frac{2\pi}{T} = \pi$$

$$\Rightarrow \quad T = 2\ s$$

459 (a)

$$mg = 2kx_A$$

$$mg = kx_B$$

$$\therefore \quad \frac{x_A}{x_B} = \frac{1}{2}$$

$$\therefore \quad W = Fx$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{Fx_A}{Fx_B} = \frac{1}{2}$$

460 (a)
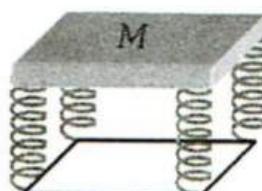
The springs are in parallel

Springs constant $= 4 \times 4000\ Nm^{-1}$

$M = 40\ kg$

$$\therefore \text{Period of oscillation, } T = 2\pi\sqrt{\frac{40}{16000}}$$

$$\Rightarrow T = \frac{2\pi}{20} = \frac{\pi}{10} \Rightarrow T = 0.314s$$

461 (b)

The relation for kinetic energy of S.H.M. is given by

$$= \frac{1}{2}m\omega^2(a^2 - x^2) \quad ...(i)$$

Potential energy is given by

$$= \frac{1}{2}m\omega^2x^2 \quad ...(ii)$$

Now for the condition of question and from eqs. (i) and (ii)

$$\frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{3} \times \frac{1}{2}m\omega^2x^2$$

or $\frac{4}{6}m\omega^2 x^2 = \frac{1}{2}m\omega^2 a^2$ or $x^2 = \frac{3}{4}a^2$

So, $x = \frac{a}{2}\sqrt{3} = 0.866\,a \approx 87\%$ of amplitude

462 (b)

$$t_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad \text{or} \quad t_1^2 = \frac{2\pi^2 m}{k_1}$$

or $k_1 = \frac{4\pi^2 m}{t_1^2}$

Similarly, $k_2 = \frac{4\pi^2 m}{t_2^2}$

And $(k_1 + k_2) = \frac{4\pi^2 m}{t_0^2}$

$\therefore \quad \frac{4\pi^2 m}{t_0^2} + \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$

or $\frac{1}{t_0^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

463 (b)

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$mv^2 = \frac{1}{2}kx^2 \Rightarrow kx^2 = 2mv^2 \Rightarrow x = \sqrt{\frac{2mv^2}{k}}$$

464 (c)

Kinetic energy $K = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2} \times 10 \times \left(\frac{2\pi}{2}\right)^2 [10^2 - 5^2] = 375\pi^2\,ergs$

465 (d)

At time $\frac{T}{2}$; $v = 0$ $\therefore$ Total energy = Potential energy

466 (d)

KE at mean position$= \frac{1}{2}m\omega^2 a^2 = 8 \times 10^{-3}$

or $\omega = \left(\frac{2 \times 8 \times 10^{-3}}{ma^2}\right)^{1/2}$

$= \left[\frac{2 \times 8 \times 10^{-3}}{0.1 \times (0.1)^2}\right]^{1/2} = 4$

Equation of SHM is,

$y = a\sin(\omega t + \theta) = 0.1\sin\left(4t + \frac{\pi}{4}\right)$

467 (d)

The maximum force acting on the body executing simple harmonic motion is

$m\omega^2 a = m \times (2\pi f)^2 a = 60 \times \left(2\pi \times \frac{2}{\pi}\right)^2 \times 0.1 N$

$= 60 \times 16 \times 0.1 = 96N = \frac{96}{9.8} \approx 10kgf$ and this force is towards mean position

Upper extreme   Equilibrium position   Lower extreme
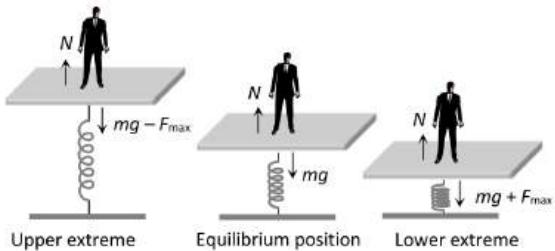
The reaction of the force on the platform away from the mean position. It reduces the weight of man on upper extreme, $i.e.$, net weight $= (60 - 10)kgf$.

This force adds to the weight at lower extreme position

$i.e.$, net weight becomes $= (60 + 10)kgf$

Therefore, the reading the weight recorded by spring balance fluctuates between $50\,kgf$ and $70\,kgf$

468 (d)

The period of pendulum doesn't depends upon mass but it depends upon length (distance between point of suspension and centre of mass) In first three cases length are same so $T = T_1 = T_2$ but in last case centre of mass lowers which in turn increases the length. So in this case time period will be more than the other cases

469 (b)

Acceleration of simple harmonic motion is

$$a_{max} = -\omega^2 A$$

or $\quad \frac{(a_{max})_1}{(a_{max})_2} = \frac{\omega_1^2}{\omega_2^2}$

(as A remains same)

or $\quad \frac{(a_{max})_1}{(a_{max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1:10^2$

470 (d)

After standing centre of mass of the oscillating body will shift upward therefore effective length will decrease and by $T \propto \sqrt{l}$, time period will decrease

471 (b)

Kinetic energy $\quad K = \frac{1}{2}m\omega^2 A^2 - y^2$

At mean position $\quad y = 0$

$$K = \frac{1}{2}m\omega^2(A^2)$$

Potential energy $\quad U = \frac{1}{2}m\omega^2 y^2$

…(i)

U at $y = \frac{A}{2}$

$$U = \frac{1}{2}\frac{mA^2}{4}\omega^2$$

…(ii)

Dividing Eq. (i) by Eq. (ii), we get
$$\frac{KE}{U} = \frac{4}{1}$$

472 (c)

The given equation is written as
$$y = a(\sin \omega t + \cos \omega t)$$
or $\quad y = a\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin \omega t + \frac{1}{\sqrt{2}}\cos \omega t\right)$

or $\quad y = a\sqrt{2}\left[\cos\frac{\pi}{4}\sin \omega t + \sin\frac{\pi}{4}\cos \omega t\right]$

or $\quad y = a\sqrt{2}\,\sin(\omega t + \frac{\pi}{4})$

Thus, we have seen that the particle's motion is simple harmonic with amplitude $a\sqrt{2}$.

473 (c)

Amplitude resonance takes place at a frequency of external force which is less than the frequency of undamped maximum vibrations. Velocity resonance takes place (*ie*, maximum energy) when frequency of external periodic force is equal to natural frequency of undamped vibrations

474 (c)

Equation of motion $y = a\cos \omega t$
$$\Rightarrow \frac{a}{2} = a\cos \omega t \Rightarrow \cos \omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$$
$$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{3} \Rightarrow t = \frac{\frac{\pi}{3} \times T}{2\pi} = \frac{4}{3 \times 2} = \frac{2}{3}s$$

475 (b)

Resolve tension T in string into two rectangular components. Then
$T\cos \theta = mg$
And $\quad T\sin \theta = mr\,\omega^2$
So $\frac{T\sin\theta}{T\cos\theta} = \tan\theta = \frac{r\omega^2}{g}$
Or $\quad g\tan\theta = r\omega^2 + r4\pi^2/T^2$
Or $\quad T = 2\pi\sqrt{\frac{r}{g\tan\theta}} \quad (\because r = l\sin\theta)$
$$= 2\pi\sqrt{\frac{l\sin\theta}{g\tan\theta}}$$
$$= 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

476 (d)

$v_{\max} = a\omega$ and $A_{\max} = a\omega^2 \Rightarrow \omega = \frac{A_{\max}}{v_{\max}} = \frac{4}{2} = 2rad/s$

477 (c)

$$\because mg = kx \Rightarrow \frac{m}{k} = \frac{x}{g} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{g}}$$

$$= 2\pi\sqrt{\frac{9.8 \times 10^{-2}}{9.8}} = \frac{2\pi}{10}s$$

478 (c)

The bob possess kinetic energy at its mean position which gets converted to potential energy at height h. But the total energy remains converted.

KE
(Mean position)
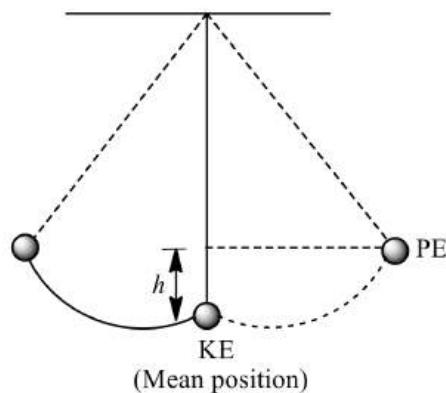
Hence, we have
$$KE = PE$$
Let velocity of the bob at mean position be v and m be its mass, then we have
$$\frac{1}{2}mv^2 = mgh$$
$$\Rightarrow \qquad v = \sqrt{2gh}$$
Putting $g = 9.8 \text{ ms}^{-2}, h = 0.1$ m
$$\therefore \qquad v = \sqrt{2 \times 9.8 \times 0.1} = 1.4 \text{ ms}^{-1}$$

479 (d)

In the given case effective acceleration $g_{eff.} = 0 \Rightarrow T = \infty$

480 (c)

Velocity in mean position $v = a\omega$, velocity at a distance of half amplitude
$$v' = \omega\sqrt{a^2 - y^2} = \omega\sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a\omega = \frac{\sqrt{3}}{2}v$$

481 (b)

$$x = a\sin\left(\omega t + \frac{\pi}{6}\right)$$
$$v = \frac{dx}{dt} = a\,\omega\cos\left(\omega t + \frac{\pi}{6}\right) \quad ...(i)$$
We know that $v_{\max} = a\omega$
So, by substituting $v = \frac{a\omega}{2}$ in equation (i) we get time (t)
$$\frac{a\omega}{2} = a\omega\cos\left(\omega t + \frac{\pi}{6}\right) \Rightarrow \frac{\pi}{3} = \omega t + \frac{\pi}{6} \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T}.t$$
$$\Rightarrow t = \frac{T}{12}$$

483 (a)

The displacement equation of particle executing SHM is

$x = a\cos(\omega t + \phi)$ ...(i)

Velocity, $v = \dfrac{dx}{dt} = -a\omega\sin(\omega t + \phi)$ ...(ii)

Acceleration,
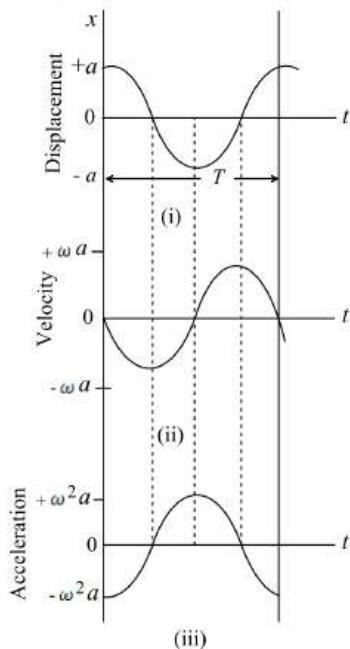
$A = \dfrac{dv}{dt} = -a\omega^2\cos(\omega t + \phi)$ ...(iii)

Fig. (i) is a plot of Eq. (i) with $\phi = 0$. Fig. (ii) shows Eq. (ii) also with $\phi = 0$. Fig. (iii) is a plot of Eq. (iii). It should be noted that in the figures the curve of v is shifted (to the left) from the curve of x by one-quarter period $(1/4T)$. Similarly, the acceleration curve of A is shifted (to the left) by $1/4T$ relative to the velocity curve of v. This implies that velocity is $90°(0.5\pi)$ out of phase with the displacement and the acceleration is $90°$ (0.5π) out of phase with the velocity but $180°$ (π) out of phase with displacement

484 (c)

$v_{\max} = a\omega = a\dfrac{2\pi}{T} = \dfrac{2\pi a}{2\pi\sqrt{\frac{m}{k}}} = a\sqrt{\dfrac{k}{m}}$

Hence, $\dfrac{v_{\max 1}}{v_{\max 2}} = \dfrac{a_1}{a_2}\sqrt{\dfrac{k_1}{k_2}}$

$\because$ $v_{\max 1} = v_{\max 2}$

(given)

$\dfrac{a_1}{a_2} = \sqrt{\dfrac{k_2}{k_1}}$

485 (d)

Sec bt is not defined for $bt = \pi/2$

$x = a\sec bt + c\csc bt = \dfrac{a\sin bt + c\cos bt}{\sin bt\cos bt}$

This equation cannot be modified in the form of simple equation of SHM ie,

$x = a\sin(\omega t + \phi)$

So, it cannot represent SHM

486 (a)

Velocity, $v = \dfrac{dx}{dt} = -A\omega\sin(\omega t + \pi/4)$

Velocity will be maximum, when

$\omega t + \pi/4 = \pi/2$

or $\omega t = \pi/2 - \pi/4 = \pi/4$ or $t = \pi/4\omega$

487 (c)

$x = A\cos\left(\omega t + \alpha + \dfrac{\pi}{2}\right)$

$= -A\sin(\omega t + \alpha)$ ...(i)

$Y = A\cos(\omega t + \alpha)$ ...(ii)

Squaring and adding Eqs. (i) and (ii), we get

$x^2 + y^2 = A^2[\sin^2(\omega t + \alpha) + \cos^2(\omega t + \alpha)] = A^2$

It is an equation of a circle. The given motion is anticlockwise

488 (a)

Work done in stretching $(W) \propto$ Stiffness of spring $(i.e.\,k)$

$\because k_A > k_B \Rightarrow W_A > W_B$ [For same extension]

489 (c)

Acceleration $= \omega^2 a$ at extreme position is maximum

490 (d)

$A = 5$ cm

$V_{\text{man}} = 3.14$ cm/s

$\Rightarrow A\omega = 31.4$

$\Rightarrow \omega = \dfrac{31.4}{5}$

$\Rightarrow 2\pi v = \dfrac{31.4}{5}$

$\Rightarrow v = \dfrac{31.4}{2\times 3.14\times 5} = 1$ Hz

492 (a)

$K = 200 N/m, F = 10N$

Let the elongation be x

$F = Kx$

$10 = 200 \times x \Rightarrow x = \dfrac{1}{20}m \Rightarrow x = 5cm$

493 (c)

$y = a\sin(\omega t - \alpha) = a\cos\left(\omega t - \alpha - \dfrac{\pi}{2}\right)$

Another equation is given $y = b\cos(\omega t - \alpha)$

So, there exists a phase difference of $\dfrac{\pi}{2} = 90°$

494 (d)

$\dfrac{1}{2}m\omega^2 r^2 = 80\,J;$

PE$= \dfrac{1}{2}m\omega^2 y^2 = \dfrac{1}{2}m\omega^2 \times \left(\dfrac{3}{4}r\right)^2$

$$= \frac{9}{16}\left(\frac{1}{2}m\omega^2 r^2\right) = \frac{9}{16}\times 80 = 45 \text{ J}$$

495 (c)

If L is the original length of spring, and k is a spring constant of the spring, then

$L + (5/k) = l$ ...(i)

and $L + (4/k) = h$ ...(ii)

$\therefore$ $l - h = 1/k$ or $k = 1/(l-h)$

and $L = (5h - l)$

$\therefore$ Length of spring when subjected to tension 9 N is

$= L + 9/k = (5h - 4l) + 9(l - h)$

$= (5l - 4h)$

496 (d)

For simple harmonic motion, $y = a\sin\ \omega t$

$\therefore$ $y = a\sin\left(\frac{2\pi}{T}\right)t$

(at $t=2$ s)

$$y_1 = a\sin\left[\left(\frac{2\pi}{16}\right)\times 2\right]$$
$$= a\sin\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}}$$

...(i)

At $t=4$ s or after 2 s from mean position.

$y_1 = \frac{a}{\sqrt{2}}$, velocity=4 ms^{-1}

$\therefore$ Velocity $= \omega\sqrt{a^2 - y_1^2}$

or $4 = \left(\frac{2\pi}{16}\right)\sqrt{a^2 - \frac{a^2}{2}}$

[From Eq. (i)]

or $4 = \frac{\pi}{8}\times\frac{a}{\sqrt{2}}$

or $a = \frac{32\sqrt{2}}{\pi}$ m

497 (b)

By using $K \propto \frac{1}{l}$

Since one fourth length is cut away so remaining length is $\frac{3}{4}th$, hence k becomes $\frac{4}{3}$ times $i.e., K' = \frac{4}{3}K$

498 (b)

Weight kept on the system will separate from the piston when the maximum force just exceeds the weight of the body. Hence, $m\omega^2 y = mg$

or $y = g/\omega^2 = 9.8/(2\pi)^2 = 0.25$ m

499 (c)

Suppose at displacement y from mean position potential energy = kinetic energy

$\Rightarrow \frac{1}{2}m(a^2 - y^2)\omega^2 = \frac{1}{2}m\omega^2 y^2$

$\Rightarrow a^2 = 2y^2 \Rightarrow y = \frac{a}{\sqrt{2}}$

500 (b)

At maximum compression the solid cylinder will stop

So loss in K.E. of cylinder = gain in P.E. of spring

$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{3}{4}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{3}{4}\times 3\times(4)^2 = \frac{1}{2}\times 200\times x^2$

$\Rightarrow \frac{36}{100} = x^2$

$\Rightarrow x = 0.6\ m$

501 (b)

$$v_{\max} = a\omega = a\frac{2\pi}{T} = 3\times\frac{2\pi}{6} = \pi\ cm/s$$

502 (c)

Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\omega^2\cos^2\omega t$

$= \frac{1}{2}m\omega^2 a^2(1 + \cos 2\omega t)$ hence kinetic energy varies periodically with double the frequency of S.H.M. $i.e., 2\omega$

503 (a)

Here, $a = 1$ cm $=0.01$ m; The mass will remain in contact with surface, if

$mg = m\omega^2 a$ or $\omega\sqrt{g/a}$

or $2\pi v = \sqrt{g/a}$ or $v = \frac{1}{2\pi}\sqrt{\frac{g}{a}}$

$= \frac{7}{2\times 22}\sqrt{\frac{980}{1}} = 4.9$ Hz $= 5$ Hz

504 (d)

In a circular motion particle repeats after equal intervals of time. So particle motion on a circular path is periodic but not simple harmonic as it does not execute to and fro motion about a fixed point.

505 (c)

Reduced mass of the system,

$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m\times 3m}{m + 3m} = \frac{3m}{4}$

Period of oscillation,

$T = 2\pi\sqrt{\frac{\mu}{k}} = 2\pi\sqrt{\frac{3m/4}{k}} = \pi\sqrt{\frac{3m}{k}}$

506 (b)

The equation for a harmonic progressive wave is

$y = 6\sin 2\pi(2t - 0.1x)$

$\Rightarrow y = 6\sin(4\pi t - 2\pi\times 0.1x)$ This is of the form

$y = A\sin(\omega t - kx)$ where $k = \frac{2\pi}{\lambda}$ $\therefore \lambda = 10mm$

The phase difference for two particles separated

by 2 mm is $\phi = \frac{2\pi}{10} \times 2 \Rightarrow \phi = \frac{2\pi}{5} = 72°$

507 (c)

Here, $y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$

$y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

$y_1 = 5\sin 2\pi t + 5\sqrt{3}\cos 2\pi t$

As of the form of $y_1 = \alpha\sin 2\pi t + \beta\cos 2\pi t$

Let $\alpha = r\cos\theta = 5, \beta = r\sin\theta = 5\sqrt{3}$

$\therefore y_1 = r\cos\theta\sin 2\pi t + r\sin\theta\cos 2\pi t$

$= r\sin(2\pi t + \theta)$

Also, $\alpha^2 + \beta^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2$

$\Rightarrow r = \sqrt{\alpha^2 + \beta^2} = \sqrt{(5)^2 + \left(5\sqrt{3}\right)^2}$

$= 5\sqrt{1^2 + \left(\sqrt{3}\right)^2} = 10$

$\therefore y_1 = 10\sin(2\pi t + \theta)$

$\therefore \frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

508 (c)

At mean position, $F_{\text{net}} = 0$

$\therefore$ By conservation of linear momentum

$Mv_1 = (M + m)v_L$

$M\omega_1 A_1 = (M + m)\omega_2 A_2$

$\omega_1 = \sqrt{\frac{k}{m}}$

$\omega_2 = \sqrt{\frac{k}{M+m}}$

On solving

$\frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$

509 (d)

In series combination

$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_2 + k_1}{k_1 k_2} \Rightarrow k_S = \frac{k_1 k_2}{k_1 + k_2}$

510 (c)

When $t = \frac{T}{12}$, then $x = A\sin\frac{2\pi}{T} \times \frac{T}{12} = \frac{A}{2}$

$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(r^2 - x^2)$

$= \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$= \frac{3}{4}\left(\frac{1}{2}m\omega^2 A^2\right)$

$\text{PE} = \frac{1}{2}m\omega^2 x^2 = \frac{1}{4}\left(\frac{1}{2}m\omega^2 A^2\right)$

$\frac{\text{KE}}{\text{PE}} = \frac{3}{1}$

OSCILLATIONS

Assertion - Reasoning Type
This section contain(s) 0 questions numbered 1 to 0. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

  a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

  b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

  c) Statement 1 is True, Statement 2 is False

  d) Statement 1 is False, Statement 2 is True

1

Statement 1: The amplitude of an oscillation pendulum decreases gradually with time

Statement 2: The frequency of the pendulum decreases with time

2

Statement 1: In simple harmonic motion, the velocity is maximum when acceleration is minimum

Statement 2: Displacement and velocity of S.H.M. differ is phase by $\pi/2$

3

Statement 1: Ocean waves hitting a beach are always found to be nearly normal to the shore

Statement 2: Ocean waves hitting a beach are assumed as plane waves

4

Statement 1: When a simple pendulum is made to oscillate on the surface of moon, its time period increases

Statement 2: Moon is much smaller as compared to earth

5

Statement 1: When a simple pendulum is made to oscillate on the surface of moon, its time period increases

Statement 2: Moon is much smaller as compared to earth

6

Statement 1: In S.H.M., the motion is 'to and fro' and periodic

Statement 2: Velocity of the particle $(v) = \omega\sqrt{k^2 - x^2}$ (where x is the displacement and k is amplitude)

7

Statement 1: Sine and cosine functions are periodic functions

Statement 2: Sinusoidal functions repeats it values after a definite interval of time

8

Statement 1: The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%

Statement 2: Time period is directly proportional to length of pendulum

9

Statement 1: The periodic time of a hard spring is less as compared to that of a soft spring

Statement 2: The periodic time depends upon the spring constant, and spring constant is large for hard spring

10

Statement 1: In extreme position of a particle executing S.H.M., both velocity and acceleration are zero

Statement 2: In S.H.M., acceleration always acts towards mean position

11

Statement 1: Consider motion for mass spring system under gravity, motion of M is not a simple harmonic motion unless Mg is negligibly small

Statement 2: For simple harmonic motion acceleration must be proportional to displacement and is directed towards the mean position
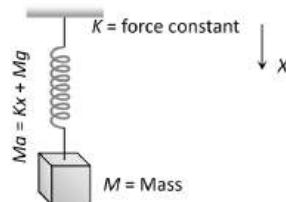
12

Statement 1: A spring of force constant k is cut into two pieces whose lengths are in the ratio 1:2. The force constant of series combination of the two parts is $\frac{\sqrt{3k}}{2}$

Statement 2: The spring connected in series are represented by $k=k_1+k_2$.

13

Statement 1: The amplitude of a particle of a particle executing SHM with frequency of 60 Hz is 0.01 m. The maximum value of acceleration of the particle is $\pm 144\pi^2 ms^{-2}$

Statement 2: Acceleration amplitude$= \omega^2 A$, where A is displacement amplitude

14

Statement 1: The amplitude of oscillation can never be infinite

Statement 2: The energy of oscillator is continuously dissipated

15

Statement 1: The graph of total energy of a particle in SHM $w.r.t.$, position is a straight line with zero slope

Statement 2: Total energy of particle in SHM remains constant throughout its motion

16

Statement 1: Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum

Statement 2: The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force

17

Statement 1: The bob of simple pendulum is a ball full of water. If a fine hole is made at the bottom of the ball. Then the time period will no more remain constant.

Statement 2: The time period of a simple pendulum does not depend upon mass.

18

Statement 1: When a girl sitting on a swing stands up, the periodic time of the swing decreases.

Statement 2: When the girl stands up more weight is felt on the string of swing.

19

Statement 1: The height of a liquid column in a u tube is 0.3m. if the liquid in one of the limbs is depressed and then released the time period of a liquid column will be 1.1s.

Statement 2: This follows from, the relation $T = 2\pi \dfrac{\sqrt{h}}{g}$

20

Statement 1: Acceleration is proportional to the displacement. This condition is not sufficient for motion in simple harmonic

Statement 2: In simple harmonic motion direction of displacement is also considered

21

Statement 1: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement 2: $k_1 < k_2$

22

Statement 1: Soldiers are asked to break steps while crossing the bridge

Statement 2: The frequency of marching may be equal to the natural frequency of bridge and may lead to resonance which can break the bridge

23

Statement 1: For an oscillating simple pendulum, the tension in the string is maximum at the mean position and minimum at the extreme position

Statement 2: The velocity of oscillating bob in simple harmonic motion is maximum at the mean position

24

Statement 1: The time period of a simple pendulum is of infinite length is infinite

Statement 2: The time period of simple pendulum is directly proportional to the square root of length.

25

Statement 1: All oscillatory motions are necessarily periodic motion but all periodic motion are not oscillatory

Statement 2: Simple pendulum is an example of oscillatory motion

26

Statement 1: Water in a U-tube executes SHM, the time period of mercury filled up to the same height in the U-tube be greater than that in case of water

Statement 2: The amplitude of an oscillating pendulum goes on increasing

27

Statement 1: In a S.H.M., kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude

Statement 2: In SHM, kinetic energy is zero when potential energy is maximum

28

Statement 1: Damped oscillation indicates loss of energy

Statement 2: The energy loss is damped oscillation may be due to friction, air resistance etc.

29

Statement 1: If the length of a spring is made n times, the spring factor of the spring becomes $1/n$ th of its original value.

Statement 2: Time of oscillation of a spring pendulum is $T = 2\pi\sqrt{\dfrac{m}{k}}$

OSCILLATIONS

: ANSWER KEY :

1) c	2) b	3) a	4) b	21) d	22) a	23) b	24) b
5) b	6) b	7) a	8) c	25) b	26) d	27) b	28) b
9) a	10) d	11) d	12) d	29) a			
13) b	14) a	15) a	16) c				
17) b	18) c	19) a	20) a				

OSCILLATIONS

: HINTS AND SOLUTIONS :

1 **(c)**
The amplitude of an oscillating pendulum decreases with time because of friction due to air.
Frequency of pendulum is independent $\left(T = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)$ of amplitude

2 **(b)**
$x = a \sin \omega t$ and $v = \frac{dx}{dt} = a\omega \cos \omega t$

It is clear phase difference between $'x'$ and $'a'$ is $\pi/2$

3 **(a)**
Waves produced on the surface of water are transverse in nature. When such waves are produced in water they spread out. Till the ocean waves reach the beach-shore, they acquire such a large radius of curvature that they may be assumed as plane waves. Hence, ocean waves hit the beach normal to the shore

4 **(b)**
$T = 2\pi\sqrt{\frac{l}{g}}$. On moon, g is much smaller compared to g on earth. Therefore, T increases

It is also true that moon is smaller than the earth, but this statement is not explaining the assertion

5 **(b)**
At the moon the value of g is less than on plane.
As $T = 2\pi\sqrt{\frac{l}{g}}$ or $T \propto \frac{1}{g}$, so T increases. It is also true that moon; is smaller than the earth, but this statement is not explaining the assertion

7 **(a)**
A periodic function is one whose value repeats after a definite interval of time. $\sin \theta$ and $\cos \theta$ are periodic functions because they repeat itself after 2π interval of time

sin curve   cos curve
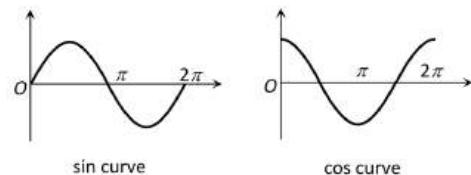
8 **(c)**
Time period of simple pendulum of length l is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T \propto \sqrt{l}$$

$$\sqrt{\frac{\Delta T}{T}} = \frac{1}{2}\frac{\Delta l}{l}$$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2} \times 3 = 1.5\%$$

9 **(a)**
The time period of a oscillating spring is given by,

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$. Since the spring constant is large for hard spring, therefore hard spring has a less periodic time as compared to soft spring

10 **(d)**
In simple harmonic motion the velocity is given by,

$v = \omega\sqrt{a^2 - y^2}$ at extreme position, $y = a$

$\therefore v = 0$

But acceleration $A = -\omega^2 a$, which is maximum at extreme position

12 **(d)**

$$k = \frac{F}{1}$$

Or $k \propto \dfrac{1}{l}$

Or $\dfrac{K_1}{K_2} = \dfrac{l_2}{L_1} = \dfrac{2}{1}$

$\because$ $K_1 = 2K, K_2 = K$

In series, $\dfrac{1}{K'} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

$= \dfrac{1}{2K} + \dfrac{1}{K} = \dfrac{3}{2K}$

$\therefore K' \dfrac{2}{3} k$

13 **(b)**

Maximum acceleration

$= \omega^2 A = 4\pi^2 v^2 A = 4\pi^2 \times (60)^2 \times 0.01$
$\qquad = 144\pi^2 \text{ ms}^{-1}$

As during SHM the direction of deflection is opposite of displacement. It may be $+ve$ or $-ve$.

Hence, maximum acceleration $= \pm 144\pi^2 \text{ms}^{-2}$

14 **(a)**

From equation, amplitude of oscillation

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$$

In absence of damping force ($b = 0$), that the steady state amplitude approaches infinity as $\omega \to \omega_0$
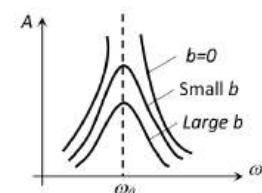

That is, if there is no resistive force in the system and then it is possible to drive an oscillator with sinusoidal force at the resonance frequency, the amplitude of motion will build up without limit. This does not occur in practice because some damping is always present in real oscillation

15 **(a)**

The total energy of S.H.M = Kinetic energy of particle + potential energy of particle

The variation of total energy of the particle in SHM with time is shown in a graph
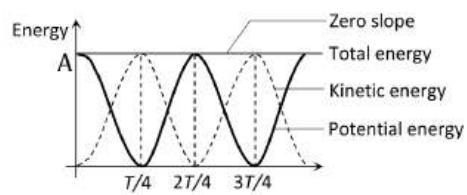
16 **(c)**

Amplitude of oscillation for a forced, damped oscillator is $A = \dfrac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2) + (b\omega/m)^2}}$, where b is constant related to the strength of the resistive force, $\omega_0 = \sqrt{k/m}$ is natural frequency of undamped oscillator ($b = 0$)

When the frequency of driving force (ω) $\approx \omega_0$, then amplitude A is very larger.

For $\omega < \omega_0$ or $\omega > \omega_0$, the amplitude decreases

18 **(c)**

Correct reason is that when the girl stands up, the centre of mass of swing-girl system moves vertically upward and hence, effective length of swing pendulum (distance of centre of mass from fixed point at the top of swing) decreases and consequently, its time period decreases

19 **(a)**

Time period of a liquid column

$T = 2\pi \dfrac{\sqrt{h}}{g}$

$= 2 \times \dfrac{22}{7} \times \sqrt{\dfrac{0.3}{9.8}} = 1.1 \text{s}$

20 **(a)**

In SHM, the acceleration is always in a direction opposite to that of the displacement $i.\,e.,$ proportional to $(-y)$

21 **(d)**

$k_1 x_1 = k_2 x_2 = F$

$$W_1 = \frac{1}{2}k_1 x_1^2 = \frac{(k_1 x_1)^2}{2k_1} = \frac{F^2}{2k_1}$$

Similarly $W_2 = \frac{F^2}{2k_2} \Rightarrow W \propto \frac{1}{k}$

$W_1 > W_2 \Rightarrow k_1 < k_2$ statement 2 is true

Statement 1 $W_1 = \frac{1}{2}k_1 x^2$

$w_2 = \frac{1}{2}k_2 x^2$

So, $W_2 > W_1$

Statement 1 is false

22 **(a)**
If the soldiers while crossing a suspended bridge march in steps, the frequency of marching steps of soldiers may match with the natural frequency of oscillations of the suspended bridge. In that situation resonance will take place, then the amplitude of oscillation of the suspended bridge will increase enormously, which may cause the collapsing of the bridge. To avoid such situations the soldiers are advised to break steps on suspended bridge

23 **(b)**
In simple pendulum, when bob is in deflection position, the tension in the spring is $T = mg\cos\theta + \frac{mv^2}{l}$. Since the value of θ is different at different positions, hence tension in the string is not constant throughout the oscillation
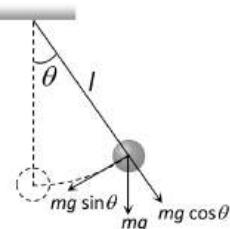
At end points θ is maximum; the value of $\cos\theta$ is least, hence the value of tension in the string is least. At the mean position, the value of $\theta = 0°$ and $\cos 0° = 1$, so the value of tension is largest

Also velocity is given by $v = \omega\sqrt{a^2 - y^2}$ which is maximum when $y = 0$, at mean position

24 **(b)**

If length of the pendulum is large, g no longer remains vertical but will be directed towards the center of the earth and expression of the time period is given by

$$T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

Here, R is the radius of earth.

If $\qquad l \to \infty, \frac{1}{l} \to 0$

And $\qquad T = 2\pi\sqrt{\frac{R}{g}} = 84.6\ min$

In general, time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{1}{g}}$$

Or $\qquad T \propto \sqrt{1}$

26 **(d)**
The period of the liquid executing SHM in a U-tube does not depend upon the density of the liquid. Therefore, time period will be the same, when mercury is filled up to the same height as the water in the U-tube.

Now, as the pendulum oscillates, it drags air along with it. Therefore, its kinetic energy is dissipated in overcoming viscous drag due to air and hence, its amplitude goes on decreasing

27 **(b)**
In SHM $K.E. = \frac{1}{2}m\omega^2(a^2 - y^2)$ and $P.E. = \frac{1}{2}m\omega^2 y^2$

For $K.E. = P.E. \Rightarrow 2y^2 = a^2 \Rightarrow y = a/\sqrt{2}$. Since total energy remains constant through out the motion, which is $E = K.E. + P.E.$ So, when $P.E.$ is maximum then $K.E.$ is zero and viceversa

28 **(b)**
Energy of damped oscillator at an any instant t is given by

$E = E_0 e^{-bt/m}$ [where $E_0 = \frac{1}{2}kx^2$ =maximum energy]

Due to damping forces the amplitude of oscillator will go on decreasing with time whose energy is expressed by above question

29 **(a)**

Correct explanation of the assertion is that spring constant of a given material and thickness is inversely proportional to its length, $ie, k \propto \frac{1}{l}$

OSCILLATIONS

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

1. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situation in Column I with the characteristics in Column II

Column-I

(A) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants

(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant

(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration

(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth Neglect forces from objects other than the earth

Column- II

(p) The object executes a simple harmonic motion

(q) The object does not change its direction

(r) The kinetic energy of the objects keeps on decreasing

(s) The object can change its direction only once

CODES :

	A	B	C	D
a)	r,q	p	q,r	p
b)	p	q,r	p	r,q
c)	q,r	p	r,q	p
d)	p	r,q	p	q,r

2. Column I gives a list of possible set of parameters measured in some experiments. The variation of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II

**Column-I**   **Column- II**

(A) Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis)   (p)
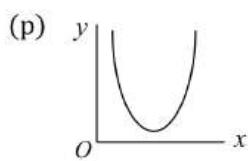
(B) Displacement (y-axis) as a function of time (x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction   (q)
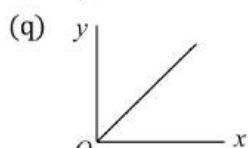
(C) Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle   (r)
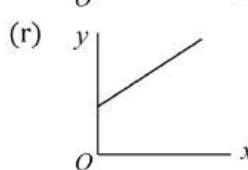
(D) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)   (s)
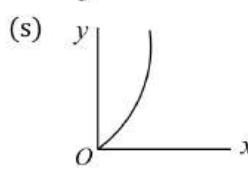
CODES :

	A	B	C	D
a)	p	q,s	s	q
b)	q,s	p	q	s
c)	s	q	q,s	p
d)	q	s	p	q,s

OSCILLATIONS

1)   **b**   **2)**   **a**

OSCILLATIONS

: HINTS AND SOLUTIONS :

2 **(a)**

(A) From nature of SHM, the graph of potential energy as function of displacement will be parabolic graph as given in option P. Hence (A) → (p)

(B) $a = 0$ or $a =$ constant. [As per given condition] $V > 0$ moving along positive x-axis

$y -$ displacement

$y = ut \pm \frac{1}{2}at^2$ for $a = $ constant

$y = ut$ for $a = 0$

These two conditions are satisfied by (q) and (s)

(B) → (q, s)

(p) is rejected because at t = 0 the displacement is not zero and velocity has negative values

(C) $R = \frac{u^2 \sin(2\theta)}{g}$ and $R \propto u^2$ for a fixed angle of projection and $u = 0, R = 0$

(C) → (s)

(D) $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2\frac{l}{g} \Rightarrow y = \frac{4\pi^2}{g}x$ (D) → (q)