12/04/2019
Morning

Answers & Solutions

for

JEE (MAIN)-2019 (Online) Phase-2

(Physics, Chemistry and Mathematics)

Time : 3 hrs. **M.M. : 360**

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage.
4. Each question is allotted 4 **(four)** marks for each correct response. *¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question.

PART–A : PHYSICS

1. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

(1) 0.24 μm (2) 0.38 μm

(3) 0.48 μm (4) 0.12 μm

Answer (1)

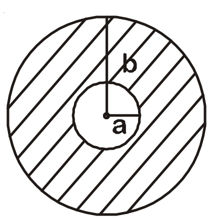
Sol. $\theta_{min} = \dfrac{1.22\lambda}{D}$

$$\frac{D}{2f} = 1.25$$

$$d_{min} = \frac{1.22\lambda f}{D} = \frac{1.22 \times 5000 \times 10^{-10}}{2.50}$$

$= 0.24\ \mu$m

2. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\dfrac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :



(1) $\dfrac{a+b}{2}$ (2) $\sqrt{\dfrac{a^2+b^2+ab}{2}}$

(3) $\dfrac{a+b}{3}$ (4) $\sqrt{\dfrac{a^2+b^2+ab}{3}}$

Answer (4)

Sol. $\Rightarrow \quad \sigma = \dfrac{\sigma_0}{r} \qquad \therefore \dfrac{\sigma_0}{r} 2\pi r dr = dm$

$\Rightarrow$ m = $\sigma_0 2\pi$ (b – a)

$$I = \sigma_0 2\pi \int_a^b r^2 dr = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$$

$$\therefore \quad mk^2 = I \Rightarrow 2\pi\sigma_0(b-a)k^2 = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$$

$$\Rightarrow \quad k^2 = \frac{1}{3}(b^2 + ab + a^2)$$

$$\therefore \quad k = \sqrt{\frac{1}{3}\frac{(b^3 - a^3)}{b-a}}$$

3. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is :

(1) $\dfrac{R}{2g}$ (2) $\dfrac{2R}{g}$

(3) $\dfrac{R}{g}$ (4) $\dfrac{R}{4g}$

Answer (2)

Sol. For same horizontal range.

$\theta_1 = \theta \qquad R = \dfrac{u^2 \sin 2\theta}{g}$

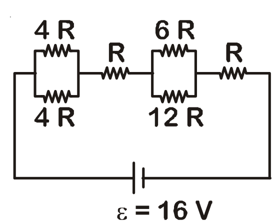
θ_2 = (90 – θ)

so $t_1 = \dfrac{2u\sin\theta}{g}$ and $t_2 = \dfrac{2u\cos\theta}{g}$

$$\therefore \quad t_1t_2 = \frac{u^2 4\sin\theta\cos\theta}{g^2}$$

$$\Rightarrow \quad t_1t_2 = \frac{2u^2 \sin 2\theta}{g^2} = \frac{2R}{g}$$

4. The resistive network shown below is connected to a D.C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is :



(1) 8 Ω (2) 1 Ω

(3) 16 Ω (4) 6 Ω

Answer (1)

Sol. $R_{eq} = 2R + R + 4R + R = 8R$

$$P = \frac{v^2}{R_{eq}} \Rightarrow \frac{16\times16}{8R} = 4 \text{ watt}$$

$$\frac{16\times16}{4\times8} = R \Rightarrow R = 8\,\Omega$$

5. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(1) $\frac{2\lambda}{(\mu-1)}$

(2) $\frac{\lambda}{2(\mu-1)}$

(3) $\frac{\lambda}{(2\mu-1)}$

(4) $\frac{\lambda}{(\mu-1)}$

Answer (4)

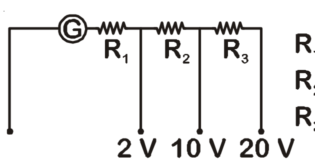
Sol. $\mu t - t = \lambda$

$\Rightarrow t(\mu - 1) = \lambda$

$$\Rightarrow t = \frac{\lambda}{(\mu-1)}$$

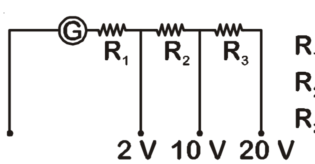
6. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is :

(1)



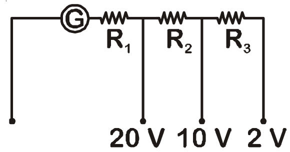
$R_1 = 1900\,\Omega$
$R_2 = 8000\,\Omega$
$R_3 = 10000\,\Omega$

(2)



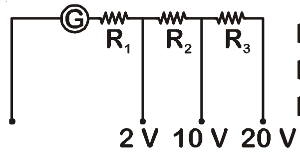
$R_1 = 1900\,\Omega$
$R_2 = 9900\,\Omega$
$R_3 = 19900\,\Omega$

(3)



$R_1 = 19900\,\Omega$
$R_2 = 9900\,\Omega$
$R_3 = 1900\,\Omega$

(4)



$R_1 = 2000\,\Omega$
$R_2 = 8000\,\Omega$
$R_3 = 10000\,\Omega$

Answer (1)

Sol. For R_1

$\because$ $I_g = 10^{-3}$ A

$\therefore$ $10^{-3}(R_1 + 100) = 2\text{ V} \Rightarrow R_1 = 1900\,\Omega$

For R_2

$10^{-3}(R_1 + R_2 + 100) = 10$ V

$\Rightarrow R_1 + R_2 + 100 = 10000$

$\Rightarrow R_2 = 8000\,\Omega$

For R_3

$10^{-3}(R_1 + R_2 + R_3 + 100) = 20$ V

$\Rightarrow R_1 + R_2 + R_3 + 100 = 20 \times 1000$

$\Rightarrow R_3 = 10000\,\Omega$

7. A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively : (Take V = 0 at infinity)

(1) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

(2) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(3) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

(4) $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

Answer (3)

Sol. $\vec{E} = K\frac{\vec{p}}{r^3}\sqrt{3\cos^2\theta + 1}$

$\Rightarrow \theta = \pi/2$ (0, d, 0)

$\therefore$ $\vec{E} = \frac{-k\vec{p}}{d^3}$

8. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by :

(1) 2.2 (2) 0.7

(3) 3.6 (4) 1.8

Answer (2)

Sol. T_1 = 2 sec., T_2 = 3/2

For place (1), $B_{H_1} = B_1 \cos 45° - \frac{B_1}{\sqrt{2}}$

For place (2), $B_{H_2} = B_2 \cos 30° = \frac{B_2\sqrt{3}}{2}$

$\therefore \quad T = 2\pi\sqrt{\frac{I}{MB_H}} \qquad \therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}}$

$\frac{4\times4}{9} = \frac{B_2\sqrt{3}\times\sqrt{2}}{2B_1} \quad \Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{6}\times9}{2\times16}$

$\Rightarrow \quad \frac{B_1}{B_2} = 0.68 \approx 0.7.$

9. An electromagnetic wave is represented by the electric field

$\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propogation $\hat{s}$, is :

(1) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$

(2) $\hat{s} = \left(\frac{3\hat{i}-4\hat{j}}{5}\right)$

(3) $\hat{s} = \left(\frac{-4\hat{k}+3\hat{j}}{5}\right)$

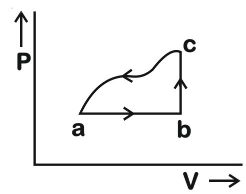
(4) $\hat{s} = \left(\frac{4\hat{j}-3\hat{k}}{5}\right)$

Answer (1)

Sol. $E = E_0 \sin(\omega t + 6y - 8z)$

$\hat{s} = \frac{8\hat{k}-6\hat{j}}{10} = \left(\frac{4\hat{k}-3\hat{j}}{5}\right)$

10. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is :



(1) 130 J (2) 100 J

(3) 140 J (4) 120 J

Answer (1)

Sol. For the process (c – a), ΔU_{ca} = – 180 J

For process (b – c) → Isochoric (W_{bc} = 0)

$\therefore$ ΔU = 60 J

Heat absorbs along (a – b), Q_{ab} = 250 J

Also $\therefore$ ΔU_{cycle} = 0

$\therefore$ ΔU_{ab} = 120 J

So $W_{a\to b}$ = 130 J

Total work done from (a → b → c)

= W_{ab} + W_{bc} = 130 J

11. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to :

(Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m², respectively ; g = 10 ms⁻²)

(1) 0.5 kg (2) 0.9 kg

(3) 1.5 kg (4) 9 kg

Answer (Bonus)

Sol. $\frac{Mgl}{A\Delta l} = Y$

$\therefore \quad \Delta l_{Mechanical} = \frac{Mgl}{AY}$

$\Delta l_{Thermal} = l\alpha\Delta T = l\alpha\times20$

$\frac{Mgl}{AY} = 20\,\alpha\,l$

$M = \frac{20\times10^{-5}\times\pi\times1\times10^{-6}\times10^{11}}{10} = 6.28\text{ kg}$

12. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2).

(1) 4×10^{-5} C (2) 3×10^{-5} C

(3) 7×10^{-6} C (4) 2×10^{-6} C

Answer (2)

Sol. $B = \frac{\mu_0 i}{2a}$ and $\frac{\omega q}{2\pi} = i$

$\therefore \quad B = \frac{\mu_0}{2a} \cdot \frac{\omega q}{2\pi}$

$B = \frac{10^{-7} \times 40}{0.1} \times q \times \pi \;\Rightarrow\; q = 3 \times 10^{-5}$ C

13. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E = \frac{1240\text{ eV}}{\lambda(\text{in nm})}$):

(1) n = 5 (2) n = 7

(3) n = 4 (4) n = 6

Answer (1)

Sol. $\Rightarrow \quad \Delta E_n = -\frac{E_0 Z^2}{n^2}$

Let it start from n to m and from m to ground.

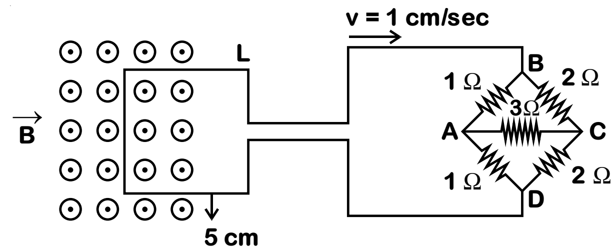
Then $13.6 \times 4 \left| 1 - \frac{1}{m^2} \right| = \frac{hc}{30.4\text{ nm}}$

$\Rightarrow \quad 1 - \frac{1}{m^2} = 0.7498 \quad \Rightarrow 0.25 = \frac{1}{m^2}$

$\therefore \quad m = 2$, and now $13.6 \times 4 \left(\frac{1}{4} - \frac{1}{n^2} \right) = \frac{hc}{108.5 \times 10^{-9}}$

$n \approx 5$.

14. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to:



(1) 170 μA (2) 60 μA

(3) 150 μA (4) 115 μA

Answer (1)

Sol. VBl = iR_{eq}

$\because \quad R_{eq} = \frac{4}{3}\,\Omega + 1.7 = 3\,\Omega$

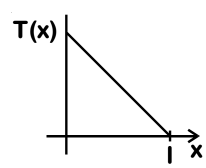
$i = \frac{(BLV)}{R_{eq}} = \frac{(1)(5 \times 10^{-2}) \times 10^{-2}}{3}$

$= \frac{5}{3} \times 10^{-4}$ A $\simeq 1.7 \times 10^{-4}$ A

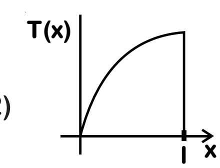
= 170 μA

15. A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

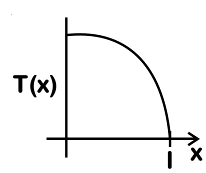
(1)



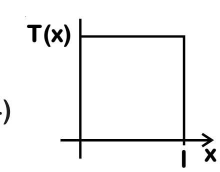
(2)



(3)

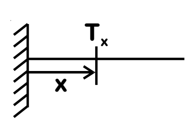


(4)



Answer (3)

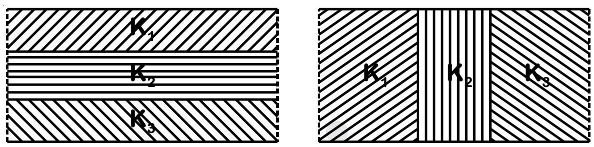
Sol.



$$T_x = \frac{M}{L}(L - x)\left\{x + \frac{L - x}{2}\right\}\omega^2 = \frac{M\omega^2}{2L}(L^2 - x^2)$$

16. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)):



(I) (II)

(1) $\dfrac{E_1}{E_2} = \dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(2) $\dfrac{E_1}{E_2} = \dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

(3) $\dfrac{E_1}{E_2} = \dfrac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(4) $\dfrac{E_1}{E_2} = \dfrac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

Answer (3)

Sol. $\dfrac{1}{C_1} = \dfrac{d}{3A\varepsilon_0}\left(\dfrac{1}{K_1}+\dfrac{1}{K_2}+\dfrac{1}{K_3}\right)$

$$C_1 = \frac{3A\varepsilon_0 (K_1K_2K_3)}{d(K_1K_2+K_2K_3+K_3K_1)}$$

$$C_2 = \frac{A\varepsilon_0}{3d}(K_1+K_2+K_3)$$

$$\frac{E_1}{E_2} = \frac{C_1}{C_2} = \frac{3K_1K_2K_3}{(K_1K_2+K_2K_3+K_3K_1)} \times \frac{3}{(K_1+K_2+K_3)}$$

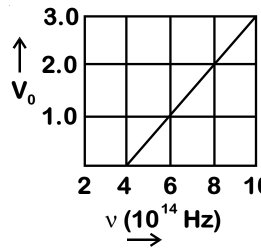
$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}$$

17. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be:

(Given: Planck's constant)

(h) = 6.63×10^{-34} Js, electron

charge e = 1.6×10^{-19} C)



(1) 1.95 eV (2) 2.12 eV

(3) 1.82 eV (4) 1.66 eV

Answer (4)

Sol. $\phi = \dfrac{hc}{\lambda} = h\nu$

$\therefore \quad \phi = h \times 4 \times 10^{14}$ Hz = 1.654 eV

$\Rightarrow \quad \phi \approx 1.66$ eV

18. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$):

(1) $\theta_0 = \sin^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$ and $v_0 = \dfrac{5}{3}ms^{-1}$

(2) $\theta_0 = \cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right)$ and $v_0 = \dfrac{3}{5}ms^{-1}$

(3) $\theta_0 = \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$ and $v_0 = \dfrac{5}{3}ms^{-1}$

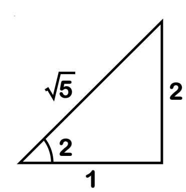
(4) $\theta_0 = \sin^{-1}\left(\dfrac{2}{\sqrt{5}}\right)$ and $v_0 = \dfrac{3}{5}ms^{-1}$

Answer (3)

Sol. $y = 2x - 9x^2$

Comparing it with equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{24^2\cos^2\theta}$$



$\therefore \quad \tan\theta = 2$

And $\quad 9 = \dfrac{10 \times 5}{2v_0^2}$

$$\Rightarrow \quad v_0 = \frac{5}{3}m/s$$

19. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to : (Speed of sound in water = 1500 ms^{-1})

(1) 499 Hz (2) 504 Hz

(3) 507 Hz (4) 502 Hz

Answer (4)

Sol. f_1 (frequency received by A)

$$= \nu_0 \left[\frac{1500-5}{1500-7.5}\right]$$

f_2 [frequency received by B]

$$= \nu_0 \times \frac{1495}{1492.5} \times \frac{1507.5}{1505}$$

= 502 Hz.

20. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to :

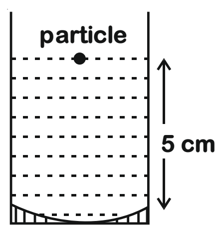
(Refractive index of water = 1.33)

(1) 11.7 cm

(2) 6.7 cm

(3) 13.4 cm

(4) 8.8 cm



Answer (4)

Sol. $$\frac{1}{V} + \frac{1}{U} = \frac{-1}{20}$$

$$\frac{1}{V} - \frac{1}{5} = \frac{-1}{20}$$

$$\frac{1}{V} = \frac{-1}{20} + \frac{1}{5}$$

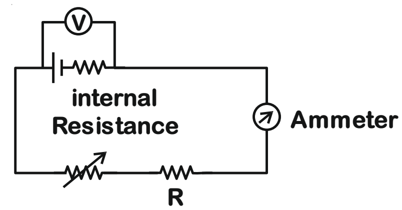
$$\frac{1}{V} = \frac{3}{20}$$

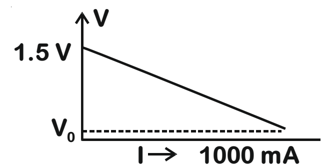
$$d = \left(\frac{20}{3} + 5\right) \times 3/4$$

$$= \frac{35}{4}$$

d = 8.8 cm

21. To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained :





If V_0 is almost zero, identify the correct statement :

(1) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(2) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

(3) The value of the resitance R is 1.5 Ω

(4) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

Answer (1)

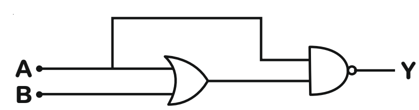
Sol. $$V = \frac{ER}{R+r}$$

for R = ∞, V = E = 1.5 V

for R = 0, I = E/r = 1

r = 1.5 Ω

22. The truth table for the circuit given in the fig. is:



(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(3)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(4)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

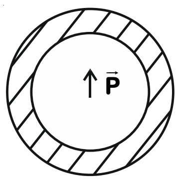
Answer (4)

Sol. $\Rightarrow\ y = \overline{\overline{A}\cdot(A+B)} = \overline{\overline{A}} + \overline{(A+B)}$

$\Rightarrow\ y = \overline{A} + \overline{A}\cdot\overline{B} = \overline{A}\,(1+\overline{B})$

$\Rightarrow\ y = \overline{A}$

23. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{P}$ as shown. In this case:



(1) Surface charge density on the outer surface depends on $|\vec{P}|$

(2) Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

(3) Electric field outside the shell is the same as that of point charge at the centre of the shell

(4) Surface charge density on the inner surface of the shell is zero everywhere

Answer (3)

Sol. Since dipole is having zero net charge. So inside surface shall have non-zero non-uniform charge distribution. And net field outside the region would be same as that would have been for point charge at surface.

24. Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

(1) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (2) $\frac{\varepsilon_0}{\mu_0}$

(3) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (4) $\frac{\mu_0}{\varepsilon_0}$

Answer (3)

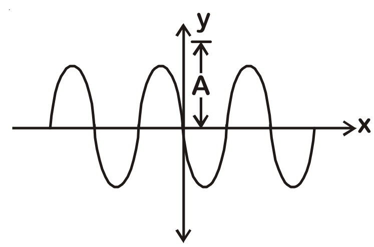
Sol. As we know t = RC

$t = \frac{L}{R}$

$R^2\frac{C}{L} = 1$

$R = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

25. A progressive wave travelling along the positive x-direction is represented by $y(x,t) = A\sin(kx-\omega t+\phi)$. Its snapshot at t = 0 is given in the figure.



For this wave, the phase ϕ is :

(1) π (2) $-\frac{\pi}{2}$

(3) $\frac{\pi}{2}$ (4) 0

Answer (1)

Sol. $y = A\sin(\omega t - kx + \phi)$

At t = 0 and x = 0 particle is at mean position and will proceed in positive y direction

26. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(1) 0.20 ms^{-1} (2) 0.14 ms^{-1}

(3) 0.47 ms^{-1} (4) 0.28 ms^{-1}

Answer (1)

Sol. $50\ V_1 = 20\ V_2$

$V_1 + V_2 = 0.70$

$V_1 = 0.20$

27. When M_1 gram of ice at –10°C (specific heat = 0.5 cal $g^{-1}°C^{-1}$) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is :

(1) $\frac{50M_2}{M_1} - 5$

(2) $\frac{5M_1}{M_2} - 50$

(3) $\frac{50M_2}{M_1}$

(4) $\frac{5M_2}{M_1} - 5$

Answer (1)

Sol. $M_1 \times 5 + M_1 L = M_2\ 50$

$$L = \frac{50M_2}{M_1} - 5$$

28. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume?

(R = 8.3 J/mol K)

(1) 19.7 J/mol K

(2) 21.6 J/mol K

(3) 15.7 J/mol K

(4) 17.4 J/mol K

Answer (4)

Sol. $5C_V = 2 \times \frac{3R}{2} + 3 \times \frac{5R}{2}$

$$C_V = \frac{21R}{10}$$

29. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l (l<<L), is close to :

(1) Mgl

(2) $Mgl\left(1+\theta_0^2\right)$

(3) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$

(4) $Mgl\left(1-\theta_0^2\right)$

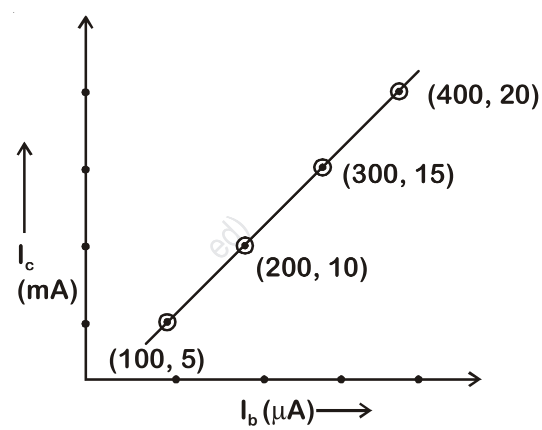
Answer (2)

Sol. $W_{man} = Mg_{eff}\ l$

$$g_{eff} = g(1+\theta_0^2)$$

$$W_{man} = Mgl(1+\theta_0^2)$$

30. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The Voltage and Power gain, are respectively :



(1) 5×10^4, 2.5×10^6

(2) 2.5×10^4, 2.5×10^6

(3) 5×10^4, 5×10^6

(4) 5×10^4, 5×10^5

Answer (1)

Sol. $\beta = \frac{I_c}{I_b}$

$$= \frac{5 \times 10^{-3}}{100 \times 10^{-6}}$$

= 50

Voltage gain $= \beta \frac{R_O}{R_i} = 5 \times 10^4$

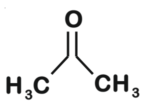
Power gain = β (voltage gain)

$= 250 \times 10^4 = 2.5 \times 10^6$

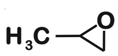
PART–B : CHEMISTRY

1. The major product of the following addition reaction is

$$H_3C-CH=CH_2 \xrightarrow{Cl_2/H_2O}$$

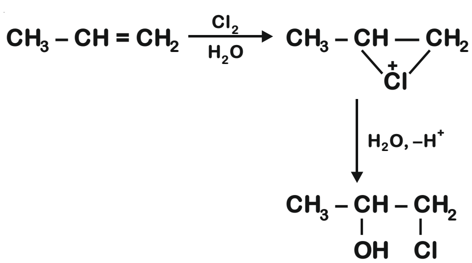
(1) 

(2) $\underset{\displaystyle\quad\ \ Cl\quad OH}{CH_3-CH-CH_2}$

(3) 

(4) $\underset{\displaystyle\quad\ \ OH\quad Cl}{H_3C-CH-CH_2}$

Answer (4)

Sol. 

2. But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give :

(1) 2 molecules of CH_3CHO

(2) 2 molecules of CH_3COOH

(3) $\underset{\displaystyle\quad\ \ OH\ \ OH}{CH_3-CH-CH-CH_3}$

(4) One molecule of CH_3CHO and one molecule of CH_3COOH

Answer (2)

Sol. $CH_3-CH=CH-CH_3 \xrightarrow[OH^-,\ \Delta]{KMnO_4} 2CH_3COOH$

3. The correct sequence of thermal stability of the following carbonates is :

(1) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

(2) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$

(3) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

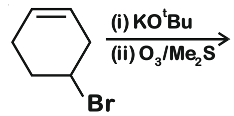
(4) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$

Answer (1)

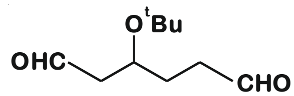
Sol. Stability of alkaline earth metal carbonates increases down the group :

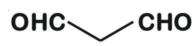
$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

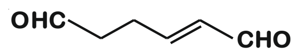
4. The major product(s) obtained in the following reaction is/are



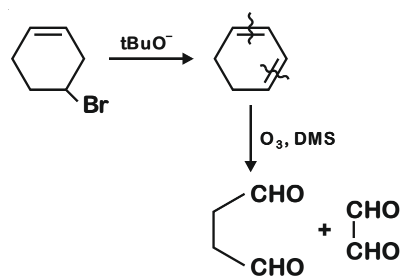
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

5. Which of the following statements is not true about RNA?

(1) It usually does not replicate

(2) It is present in the nucleus of the cell

(3) It controls the synthesis of protein

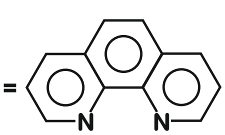
(4) It has always double stranded α-helix structure

Answer (4)

Sol. RNA has a single helix structure.

DNA has a double helix structure.

6. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is :

(Phen = 

and
ignore pairing energy)

(1) $[Ni(phen)_3]^{2+}$ (2) $[Co(phen)_3]^{2+}$

(3) $[Zn(phen)_3]^{2+}$ (4) $[Fe(phen)_3]^{2+}$

Answer (4)

Sol. $\underset{t_{2g}^6 e_g^2}{Ni^{2+}(d^8)} \longrightarrow \underset{t_{2g}^6 e_g^1}{Ni^{3+}(d^7)}$

$\underset{t_{2g}^6 e_g^1}{Co^{2+}(d^7)} \longrightarrow \underset{t_{2g} e_g^0}{Co^{3+}(d^6)}$

$\underset{t_{2g}^6 e_g^4}{Zn^{2+}(d^{10})} \longrightarrow \underset{t_{2g}^6 e_g^3}{Zn^{2+}(d^9)}$

$\underset{t_{2g}^6 e_g^0}{Fe^{2+}(d^6)} \longrightarrow \underset{t_{2g}^5 e_g^0}{Fe^{3+}(d^5)}$

So, only Fe^{2+} will lose crystal field stabilisation upon oxidation to +3, others will gain crystal field stabilisation

7. An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is :

(1) $\frac{3}{2}a$ (2) $\frac{a}{2}$

(3) a (4) $\sqrt{2}a$

Answer (2)

Sol. In FCC, tetrahedral voids are located on the body diagonal at a distance of $\frac{\sqrt{3}a}{4}$ from the corner. Together they form a smaller cube of edge length $\frac{a}{2}$.

8. Glucose and Galactose are having identical configuration in all the positions except position.

(1) C – 2 (2) C – 5

(3) C – 3 (4) C – 4

Answer (4)

Sol. Galactose and Glucose are C_4 epimers.

9. The metal that gives hydrogen gas upon treatment with both acid as well as base is :

(1) Zinc (2) Magnesium

(3) Iron (4) Mercury

Answer (1)

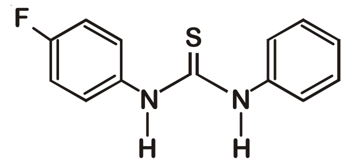
Sol. $Zn + NaOH \longrightarrow Na_2ZnO_2 + H_2$

$Zn + H_2SO_4 \longrightarrow ZnSO_4 + H_2$

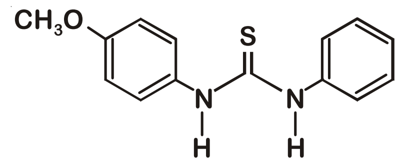
Zn is amphoteric.

10. The increasing order of the pK_b of the following compound is :

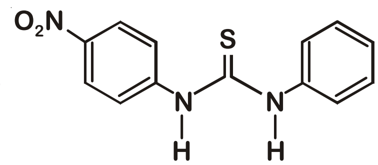
(A)



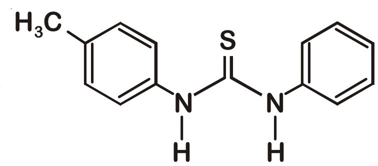
(B)



(C)



(D)



(1) (B) < (D) < (A) < (C)

(2) (A) < (C) < (D) < (B)

(3) (B) < (D) < (C) < (A)

(4) (C) < (A) < (D) < (B)

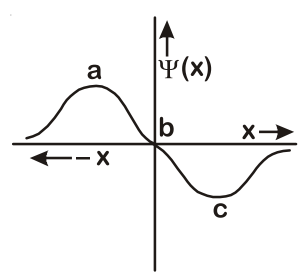
Answer (1)

Sol. EWG attached to benzene ring will reduce the basic strength and increase pK_b while EDG decreases pK_b.

Correct order of pK_b

(C) > (A) > (D) > (B)

11. The electrons are more likely to be found :



(1) In the region a and c

(2) Only in the region c

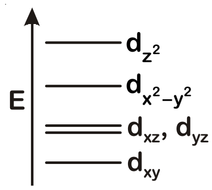
(3) Only in the region a

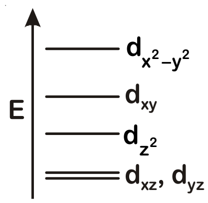
(4) In the region a and b

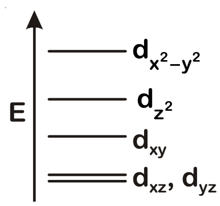
Answer (1)

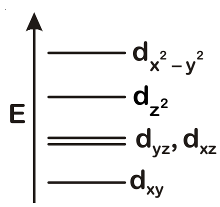
Sol. Probability of finding an electron is given by $4\pi r^2 dr\ \Psi^2$ and it will have maximum value at both ‘a’ and ‘c’.

12. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).

(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. The field becomes square planar and the order of energy is $d_{x^2-y^2} > d_{xy} > d_{z^2} > d_{zx} = d_{yz}$.

13. An example of a disproportionation reaction is:

(1) $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

(2) $2CuBr \rightarrow CuBr_2 + Cu$

(3) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

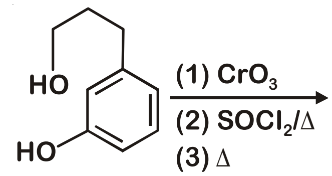
(4) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$

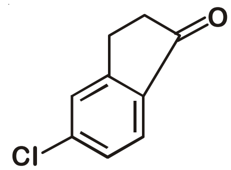
Answer (2)

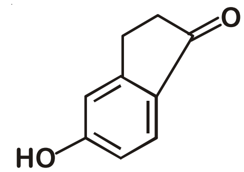
Sol. $\underset{Cu^+}{CuBr} \longrightarrow \underset{Cu^0}{Cu} + \underset{Cu^{2+}}{CuBr_2}$

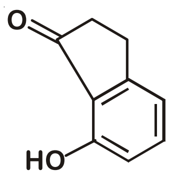
It is an example of disproportionation reaction.

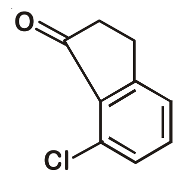
14. The major product of the following reaction is



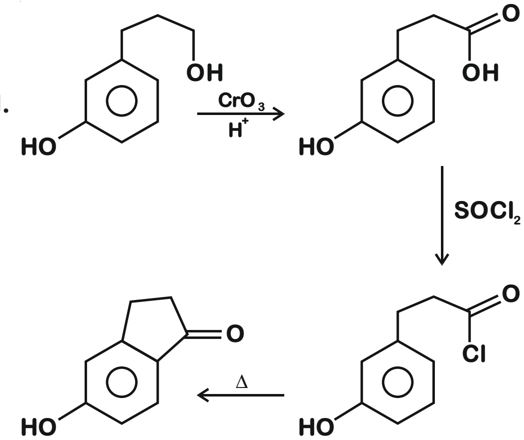
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

15. An organic compound ‘A’ is oxidized with Na_2O_2 followed by boiling with HNO_3. The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate

Based on above observation, the element present in the given compound is :

(1) Fluorine

(2) Nitrogen

(3) Phosphorus

(4) Sulphur

Answer (3)

Sol. Phosphorus is detected in the form of canary yellow ppt on reaction with ammonium molybdate.

16. The basic structural unit of feldspar, zeolites, mica, and asbestos is :

(1) $(SiO_4)^{4-}$

(2) $\left(\overset{\displaystyle R}{\underset{\displaystyle R}{\overset{|}{\underset{|}{Si}}}} - O \right)_n$ (R = Me)

(3) SiO_2

(4) $(SiO_3)^{2-}$

Answer (1)

Sol. These are examples of silicates, the basic unit being SiO_4^{4-} in each of them.

17. The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality (in mol kg^{-1}) of the aqueous solution is :

(1) 13.88×10^{-2} (2) 13.88×10^{-3}

(3) 13.88 (4) 13.88×10^{-1}

Answer (3)

Sol. Let, total 1 moles be present

$n_{solute} = 0.2$

$n_{solvent} = 0.8 \Rightarrow g_{solvent} = 0.8 \times 18$

$$\therefore \quad m = \frac{0.2 \times 1000}{0.8 \times 18}$$

$$= \frac{1000}{4 \times 18} \approx 13.88$$

18. The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are :

(1) 15, 5 and 3 (2) 15, 6 and 2

(3) 16, 5 and 2 (4) 16, 6 and 3

Answer (1)

Sol. Phosphorus has atomic number equal to 15. Its group number is 15, it has 5 valence electrons and valency equal to 3.

19. What is the molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution? Given that, solubility product of $Al(OH)_3 = 2.4 \times 10^{-24}$:

(1) 3×10^{-19}

(2) 12×10^{-21}

(3) 12×10^{-23}

(4) 3×10^{-22}

Answer (4)

Sol. $$Al(OH)_3 \rightleftharpoons \underset{s}{Al^{3+}} + \underset{\substack{0.2+3s \\ \approx 0.2}}{3OH^-}, K_{sp} = 2.4 \times 10^{-24}$$

$s(0.2)^3 = 2.4 \times 10^{-24}$

$$s = \frac{24 \times 10^{-25}}{8 \times 10^{-3}} = 3 \times 10^{-22} \frac{mol}{L}$$

20. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1 bar. The work done in kJ is :

(1) –9.0 (2) –0.9

(3) –2.0 (4) +10.0

Answer (2)

Sol. $w = -P\Delta V$

$= -(1 \text{ bar}) \times (9 \text{ L})$

$= -(10^5 \text{ Pa}) \times (9 \times 10^{-3}) \text{ m}^3$

$= -9 \times 10^2$ N-m

$= -900$ J

$= -0.9$ kJ

21. The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are :

(1) Fisher woman and concentration

(2) Washer woman and concentration

(3) Washer man and reduction

(4) Fisher man and reduction

Answer (2)

Sol. Froth floatation is a method of concentration and it was discovered by a washer women.

22. Which of the following is a thermosetting polymer?

(1) PVC (2) Buna-N

(3) Bakelite (4) Nylon 6

Answer (3)

Sol. Bakelite is an example of thermosetting polymer.

23. Peptization is a :

(1) Process of converting a colloidal solution into precipitate

(2) Process of converting precipitate into colloidal solution

(3) Process of converting soluble particles to form colloidal solution

(4) Process of bringing colloidal molecule into solution

Answer (2)

Sol. Peptisation is the process of converting a precipitate into a colloidal sol by shaking it with dispersion medium in the presence of small amount of electrolyte.

24. The correct set of species responsible for the photochemical smog is :

(1) CO_2, NO_2, SO_2 and hydrocarbons

(2) N_2, O_2, O_3 and hydrocarbons

(3) NO, NO_2, O_3 and hydrocarbons

(4) N_2, NO_2 and hydrocarbons

Answer (3)

Sol. Photochemical smog contains oxides of nitrogen, ozone and hydrocarbons.

25. Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200°C. If specific heat of I_2(s) and I_2(vap) are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250°C in cal g^{-1} is :

(1) 11.4 (2) 2.85

(3) 5.7 (4) 22.8

Answer (4)

Sol. $I_2(s) \rightarrow I_2(g)$

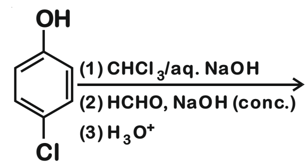
$$(\Delta H)_{T_2} - (\Delta H)_{T_1} = (\Delta C_P)(T_2 - T_1)$$

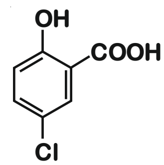
$$\therefore \quad (\Delta H)_{250} = (\Delta H)_{200} + (0.031 - 0.055)\ 50$$

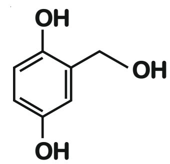
$$= 24 - 50 \times 0.024$$

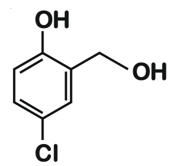
$$= 22.8$$

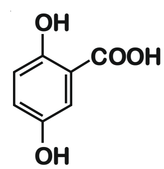
26. The major products of the following reaction are :



(1)  and Methanol

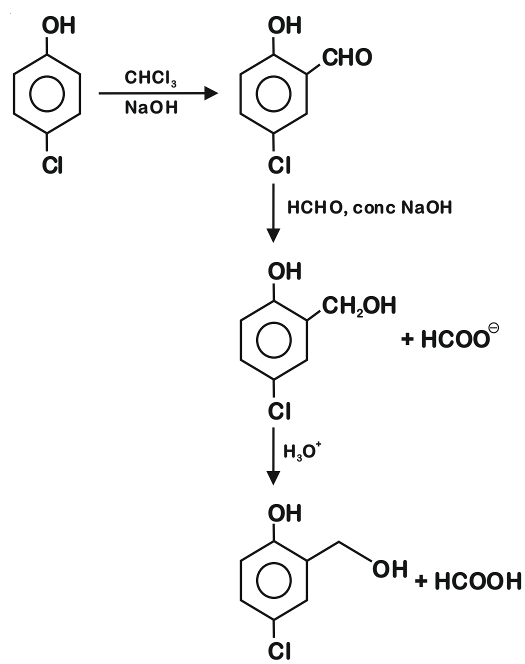
(2)  and Formic acid

(3)  and Formic acid

(4)  and Methanol

Answer (3)

Sol.



27. Given :

$Co^{3+} + e^- \rightarrow Co^{2+}$; E° = +1.81 V

$Pb^{4+} + 2e^- \rightarrow Pb^{2+}$; E° = +1.67 V

$Ce^{4+} + e^- \rightarrow Ce^{3+}$; E° = +1.61 V

$Bi^{3+} + 3e^- \rightarrow Bi$; E° = +0.20 V

Oxidizing power of the species will increase in the order :

(1) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$

(2) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$

(3) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$

(4) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

Answer (4)

Sol. Greater the reduction potential, greater is the oxidising power.

So, $Co^{3+} > Pb^{4+} > Ce^{4+} > Bi^{3+}$

28. The correct statement among the following is :

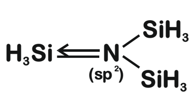
(1) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

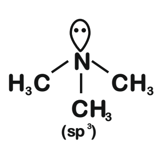
(2) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$

(3) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$

(4) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$

Answer (1)

Sol. 



Trisilylamine is planar, due to backbonding of lone pairs of nitrogen into vacant d-orbitals of Si. In trimethylamine, there is no such delocalisation and hence it is more basic.

29. In the following reaction; $xA \rightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be :

(1) C_2H_4 and C_4H_8

(2) N_2O_4 and NO_2

(3) n-Butane and Iso-butane

(4) C_2H_2 and C_6H_6

Answer (1)

Sol. $xA \rightarrow yB$

$$\therefore \quad \frac{-dA}{xdt} = \frac{1}{y}\frac{dB}{dt}$$

$$\frac{-dA}{dt} = \frac{dB}{dt} \times \frac{x}{y}$$

$$\log\left[\frac{-dA}{dt}\right] = \log\left[\frac{dB}{dt}\right] + \log\left(\frac{x}{y}\right)$$

$$\frac{x}{y} = 2$$

The reaction is of type $2A \rightarrow B$.

30. 5 moles of AB_2 weigh 125×10^{-3} kg and 10 moles of A_2B_2 weigh 300×10^{-3} kg. The molar mass of $A(M_A)$ and molar mass of $B(M_B)$ in kg mol^{-1} are :

(1) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$

(2) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$

(3) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$

(4) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$

Answer (3)

Sol. 5 mol AB_2 weighs 125 g

$\therefore$ AB_2 = 25 g/mol

10 mol A_2B_2 weighs 300 g

$\therefore$ A_2B_2 = 30 g/mol

$\therefore$ Molar mass of A = 5

Molar mass of B = 10

PART–C : MATHEMATICS

1. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix}4 & -2\\1 & -4\end{bmatrix}$ (2) $\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

(3) $\begin{bmatrix}-4 & -2\\-1 & 4\end{bmatrix}$ (4) $\begin{bmatrix}-4 & 2\\1 & 4\end{bmatrix}$

Answer (2)

Sol. Let $A=\begin{bmatrix}a & c\\c & b\end{bmatrix}$ and $B=\begin{bmatrix}0 & d\\-d & 0\end{bmatrix}$

$\Rightarrow A+B=\begin{bmatrix}a & c+d\\c-d & b\end{bmatrix}=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$

$\Rightarrow$ a = 2, b = –1, c – d = 5, c + d = 3

$\Rightarrow$ a = 2, b = –1, c = 4, d = –1

$\Rightarrow AB=\begin{bmatrix}2 & 4\\4 & -1\end{bmatrix}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

$=\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

2. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r+\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to :

(1) $\frac{21}{346}$

(2) $\frac{7}{116}$

(3) $\frac{29}{358}$

(4) $\frac{1}{12}$

Answer (4)

Sol. $375x^2 - 25x - 2 = 0$

$\alpha+\beta=\frac{25}{375},\ \alpha\beta=\frac{-2}{375}$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\alpha^r+\beta^r\right)=\left(\alpha+\alpha^2+\alpha^3+\ldots\infty\right)+\left(\beta+\beta^2+\beta^3+\ldots\infty\right)$$

$$=\frac{\alpha}{1-\alpha}+\frac{\beta}{1-\beta}$$

$$=\frac{\alpha+\beta-2\alpha\beta}{1-(\alpha+\beta)+\alpha\beta}$$

$$=\frac{\frac{25}{375}+\frac{4}{375}}{1-\frac{25}{375}-\frac{2}{375}}=\frac{29}{375-25-2}$$

$$=\frac{29}{348}=\frac{1}{12}$$

3. If $B=\begin{bmatrix}5 & 2\alpha & 1\\0 & 2 & 1\\\alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all value of α for which det (A) + 1 = 0, is :

(1) –1 (2) 2

(3) 0 (4) 1

Answer (4)

Sol. As $B = A^{-1}$

$|B|=\frac{1}{|A|}$

Now $|B|=\begin{vmatrix}5 & 2\alpha & 1\\0 & 2 & 1\\\alpha & 3 & -1\end{vmatrix}=2\alpha^2-2\alpha-25$

Given |A| + 1 = 0

$$\frac{1}{2\alpha^2-2\alpha-25}+1=0$$

$$\Rightarrow \frac{2\alpha^2-2\alpha-24}{2\alpha^2-2\alpha-25}=0$$

α = 4, –3

Sum of values = 1

4. For $x \in R$, let [x] denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots.+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is:

(1) – 135 (2) –153

(3) –133 (4) –131

Answer (3)

Sol. As $[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]....\left[x+\frac{n-1}{n}\right]=[nx]$

As [x] + [–x] = –1 ($x \notin z$)

Required value

$$=-100-\left\{\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{100}\right]+...\left[\frac{1}{3}+\frac{99}{100}\right]\right\}$$

$$=-100-\left[\frac{100}{3}\right]$$

= –133

5. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(4, 3\sqrt{2})$ (2) $(3, 3\sqrt{3})$

(3) $(5, 3\sqrt{6})$ (4) $(4, 3\sqrt{3})$

Answer (4)

Sol. $f(x)=x\sqrt{kx-x^2}=\sqrt{kx^3-x^4}$

$$f'(x)=\frac{(3kx^2-4x^3)}{2\sqrt{kx^3-x^4}}\geq 0 \text{ for } x\in[0,3]$$

$\Rightarrow$ $3k - 4x \geq 0$

$3k \geq 4x$

$3k \geq 4x$ for $x \in [0, 3]$

Hence $k \geq 4$

i.e., m = 4

For k = 4,

$\Rightarrow$ $f(x)=x\sqrt{4x-x^2}$

For max. value, f′(x) = 0

$\Rightarrow$ x = 3

i.e., $y=3\sqrt{3}$

Hence $M=3\sqrt{3}$

6. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to:

(1) $2\sqrt{14}$ (2) 14

(3) $2\sqrt{7}$ (4) $\sqrt{14}$

Answer (1)

Sol. Let P(3λ + 2, 2λ – 1, –λ + 1) and

Q(3μ + 2, 2μ – 1, –μ + 1)

As P lies on 2x + 3y – z + 13 = 0

6λ + 4 + 6λ – 3 + λ – 1 + 13 = 0

$\Rightarrow$ 13λ = –13

$\Rightarrow$ λ = –1

$\therefore$ P(–1, –3, 2)

Q lies on 3x + y + 4z = 16

9μ + 6 + 2μ – 1 – 4μ + 4 = 16

$\Rightarrow$ 7μ = 7

$\Rightarrow$ μ = 1

Q is (5, 1, 0)

$PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

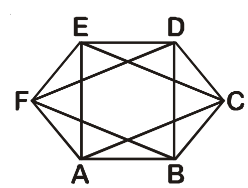
7. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is:

(1) $\frac{3}{20}$ (2) $\frac{1}{5}$

(3) $\frac{3}{10}$ (4) $\frac{1}{10}$

Answer (4)

Sol. Only two equilateral triangles are possible i.e. ΔAEC and ΔBDF.



Hence, required probability

$$=\frac{2}{^6C_3}=\frac{1}{10}$$

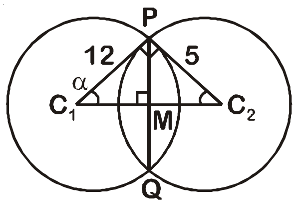
8. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is:

(1) $\frac{120}{13}$ (2) $\frac{13}{2}$

(3) $\frac{13}{5}$ (4) $\frac{60}{13}$

Answer (1)

Sol.



In ΔPC_1C_2,

$\tan\alpha = \frac{5}{12}$

$\Rightarrow \quad \sin\alpha = \frac{5}{13}$

In ΔPC_1M, $\sin\alpha = \frac{PM}{12}$

$\Rightarrow \quad \frac{5}{13} = \frac{PM}{12}$

$\Rightarrow \quad PM = \frac{60}{13}$

Length of common chord $(PQ) = \frac{120}{13}$

9. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}, \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to :

(1) $\frac{1}{\sqrt{3}}$ (2) $-\sqrt{3}$

(3) $\sqrt{3}$ (4) $-\frac{1}{\sqrt{3}}$

Answer (1*) Vector are coplanar for $\lambda = \lambda_1$ where $\lambda_1^3 - \lambda_1 + 1 = 0 \Rightarrow$ volume is minimum when $\lambda = \lambda_1$.

Sol. $V = \left\| \begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix} \right\|$

$= |1(1) + \lambda(\lambda^2) + 1(-\lambda)|$

$= |\lambda^3 - \lambda + 1|$

Let $f(x) = x^3 - x + 1$

$f'(x) = 3x^2 - 1$

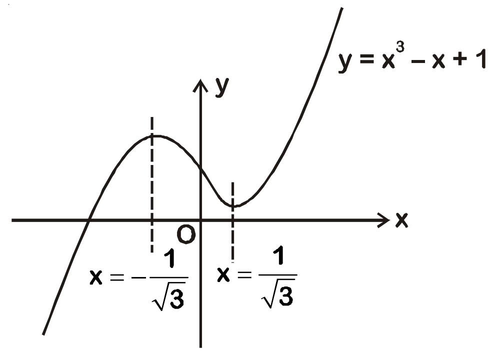
For maxima/minima, $f'(x) = 0$

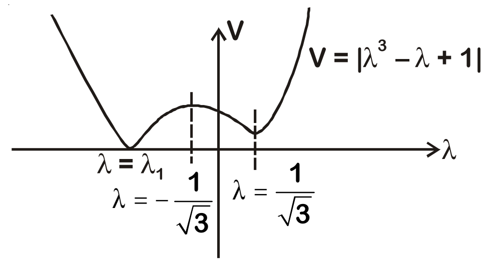
$x = \pm\frac{1}{\sqrt{3}}$

$f''(x) = 6x$

$\because \quad f''\left(\frac{1}{\sqrt{3}}\right) > 0$

$x = \frac{1}{\sqrt{3}}$ is point of local minima





When $\lambda = \lambda_1$, volume of parallelopiped is zero (vectors are coplanar)

10. The integral $\int \frac{2x^3 - 1}{x^4 + x}dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e\left|\frac{x^3+1}{x}\right| + C$ (2) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|} + C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2} + C$ (4) $\log_e\frac{|x^3+1|}{x^2} + C$

Answer (1)

Sol. $I = \int\frac{(2x^3-1)dx}{x^4+x} = \int\frac{(2x-x^{-2})dx}{x^2+x^{-1}}$

Put $x^2 + x^{-1} = t$

$(2x - x^{-2})dx = dt$

$I = \int\frac{dt}{t} = \ln|t| + c$

$= \ln|x^2 + x^{-1}| + c$

$= \ln\left|\frac{x^3+1}{x}\right| + c$

11. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{5\sqrt{5}}{2}$

(3) $\frac{\sqrt{157}}{2}$ (4) $\frac{\sqrt{221}}{2}$

Answer (2)

Sol. Slope of tangent at point P is $\frac{1}{2}$

$$3x^2 + 4y^2 = 12 \quad \Rightarrow \frac{x^2}{2^2}+\frac{y^2}{(\sqrt{3})^2}=1$$

Let point $P(2\cos\theta, \sqrt{3}\sin\theta)$

$\Rightarrow$ Equation of tangent at P is

$$\frac{x}{2}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$$

$$\Rightarrow m_T=-\frac{\sqrt{3}}{2}\cot\theta=\frac{1}{2}$$

$$\tan\theta=-\sqrt{3} \quad \Rightarrow \theta=\pi-\frac{\pi}{3} \quad \text{or } \theta=2\pi-\frac{\pi}{3}$$

If $\theta=\frac{2\pi}{3}$, then $P\left(-1,\frac{3}{2}\right)$ and $PQ=\frac{5\sqrt{5}}{2}$

If $\theta=\frac{5\pi}{3}$, then tangent does not pass through Q(4, 4)

12. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) –260 (2) –380

(3) –320 (4) –410

Answer (3)

Sol. $S_4 = 16, \quad S_6 = -48$

$2(2a + 3d) = 16$

$\Rightarrow 2a + 3d = 8$

Also $3[2a + 5d] = -48$

$\Rightarrow 2a + 5d = -16$

$2d = -24$

$d = -12 \quad \Rightarrow a = 22$

$S_{10} = 5(44 + 9(-12))$

$= -320$

13. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

(1) $\frac{25}{3}$ (2) $25\sqrt{3}$

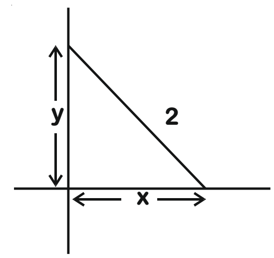
(3) $\frac{25}{\sqrt{3}}$ (4) 25

Answer (3)

Sol. Given $\frac{dy}{dt}=-25$ at y = 1

$x^2 + y^2 = 4$

When y = 1, $\quad x=\sqrt{3}$



$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$\Rightarrow x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$\Rightarrow \sqrt{3}\frac{dx}{dt}+(-25)=0$$

$$\Rightarrow \frac{dx}{dt}=\frac{25}{\sqrt{3}} \text{ cm/s}$$

14. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :

(1) $2^{20} + 1$ (2) 2^{21}

(3) $2^{20} - 1$ (4) 2^{20}

Answer (4)

Sol. Number of ways of selecting 10 objects

= (10I, 0D) or (9I, 1D) or (8I, 1D) or ... (0I, 10D)

where D signifies distinct object and I indicates identical object

$= 1 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}$

$$=\frac{2^{21}}{2}=2^{20}$$

15. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S′ denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS′ in a ratio :

(1) 13 : 11 (2) 14 : 13

(3) 5 : 4 (4) 2 : 1

Answer (3)

Sol. Equation of tangent to $y^2 = 12x$ is $y = mx + \frac{3}{m}$

Equation of tangent $\frac{x^2}{1} - \frac{y^2}{8} = 1$ is $y = mx \pm \sqrt{m^2 - 8}$

for common tangent,

$\frac{3}{m} = \pm\sqrt{m^2 - 8} \quad \Rightarrow \frac{9}{m^2} = m^2 - 8$

Put $m^2 = t$

$t^2 - 8t - 9 = 0 \quad \Rightarrow t^2 - 9t + t - 9 = 0$

$\Rightarrow (t + 1)(t - 9) = 0$

$\because \; t = m^2 \geq 0 \quad \Rightarrow t = m^2 = 9$

$\Rightarrow m = \pm 3$

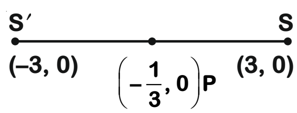
$\Rightarrow$ Equation of tangent is $y = 3x + 1$

or $y = -3x - 1$

Intersection point $P\left(-\frac{1}{3}, 0\right)$

$8 = 1(e^2 - 1) \quad \Rightarrow e = 3$

foci $(\pm 3, 0)$ $\Rightarrow$



$$\frac{S'P}{SP} = \frac{3 - \frac{1}{3}}{3 + \frac{1}{3}} = \frac{8}{10} = \frac{4}{5}$$

16. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to :

(1) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (2) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

(3) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

Answer (3)

Sol. $e^y + xy = e$ ...(i)

Put x = 0 in (i)

$\Rightarrow$ $e^y = e \Rightarrow y = 1$

Differentiate (i) w.r.t. to x

$e^y \frac{dy}{dx} + x\frac{dy}{dx} + y = 0$ ...(ii)

Put y = 1 in (ii)

$e\frac{dy}{dx} + 0 + 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{e}$

Differentiate (ii) w.r.t. to x

$e^y \frac{d^2y}{dx^2} + \frac{dy}{dx}.e^y \frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$ ...(iii)

Put y = 1, x = 0, $\frac{dy}{dx} = -\frac{1}{e}$

$e\frac{d^2y}{dx^2} + \frac{1}{e} - \frac{2}{e} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{e^2}$

$\Rightarrow \left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right) \equiv \left(-\frac{1}{e}, \frac{1}{e^2}\right)$

17. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) $2\sqrt{2}$ (2) 4

(3) 2 (4) $\sqrt{2}$

Answer (3)

Sol. $\frac{x_1 + x_2 + x_3 + x_4}{4} = 11$ and $x_1 + x_2 + x_3 + x_4 = 44$

$\frac{x_5 + x_6 + ... + x_{10}}{6} = 16 \Rightarrow x_5 + x_6 + ... + x_{10} = 96$

$x_1^2 + x_2^2 + ... + x_{10}^2 = 2000$

$\sigma^2 = \frac{\Sigma x_i^2}{N} - (\bar{x})^2$

$= \frac{2000}{10} - \left(\frac{140}{10}\right)^2 = 4$

$\Rightarrow \sigma = 2$

18. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is :

(1) 84 (2) –126

(3) –84 (4) 126

Answer (1)

Sol. $(1-x)^{10}(1+x+x^2)^9(1+x)$

$= (1-x^3)^9(1-x^2)$

$= (1-x^3)^9 - x^2(1-x^3)^9$

$\Rightarrow$ Coefficient of x^{18} in $(1-x^3)^9$ – coeff. of x^{16} in $(1-x^3)^9$.

$= {}^9C_6 = \frac{9!}{6!3!} = \frac{7 \times 8 \times 9}{6} = 84$

19. If the truth value of the statement $p \rightarrow (\sim q \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

(1) F, T, T (2) T, T, F

(3) T, F, F (4) T, F, T

Answer (2)

Sol. $P \rightarrow (\sim q \vee r)$ is a fallacy

$\Rightarrow$ P is True and $\sim q \vee r$ is False

$\Rightarrow$ P is True and $\sim q$ is False and r is False

$\Rightarrow$ Truth values of p, q, r are

T, T, F respectively.

20. Let a random variable X have a binomial distribution with mean 8 and variance 4.

If $P(X \leq 2) = \frac{k}{2^{16}}$, then k is equal to :

(1) 121 (2) 1

(3) 17 (4) 137

Answer (4)

Sol. $\mu = 8,\ \sigma^2 = 4$

$\Rightarrow\ \mu = np = 8,\ \sigma^2 = npq = 4,\ p + q = 1$

$\Rightarrow\ q = \frac{1}{2}, p = \frac{1}{2}, n = 16$

$$P(X \leq 2) = \frac{k}{2^{16}}$$

$${}^{16}C_0\left(\frac{1}{2}\right)^{16} + {}^{16}C_1\left(\frac{1}{2}\right)^{16} + {}^{16}C_2\left(\frac{1}{2}\right)^{16} = \frac{k}{2^{16}}$$

$\Rightarrow\ k = (1 + 16 + 120) = 137$

21. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is :

(1) $4\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)$ (2) $4\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

(3) $4\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ (4) $4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Answer (4)

Sol. Let vector be $\lambda\left[\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right]$

$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}$$

$$\vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

vector $= \lambda\left[\left(4\hat{i} + 4\hat{j}\right) \times \left(2\hat{i} + 4\hat{k}\right)\right]$

$= \lambda\left[16\hat{i} - 16\hat{j} - 8\hat{k}\right]$

$= 8\lambda\left[2\hat{i} - 2\hat{j} - \hat{k}\right]$

$\Rightarrow\ 12 = 8|\lambda|\sqrt{4 + 4 + 1}$

$|\lambda| = \frac{1}{2}$

Hence required vector is $\pm 4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

22. Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when x = 1, then the value of x for which y = 2, is :

(1) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (2) $\frac{3}{2} - \sqrt{e}$

(3) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (4) $\frac{1}{2} + \frac{1}{\sqrt{e}}$

Answer (3)

Sol. $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$

$$\frac{dx}{dy} + \left(\frac{1}{y^2}\right)x = \frac{1}{y^3}$$

I.F. $= e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$

its solution is

$$x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \frac{1}{y^3} dy + c$$

put $-\frac{1}{y} = t \Rightarrow \frac{1}{y^2} dy = dt$

$\Rightarrow\ x.e^{-\frac{1}{y}} = -\int te^t dt + c = -te^t + e^t + c$

$x.e^{-\frac{1}{y}} = e^{-\frac{1}{y}}\left(\frac{1}{y} + 1\right) + c$ passes through (1, 1)

$\Rightarrow\ 1 = 2 + ce \Rightarrow c = -\frac{1}{e}$

$\Rightarrow\ x = \left(1 + \frac{1}{y}\right) - \frac{1}{e}e^{\frac{1}{y}}$ passes through (k, 2)

$\Rightarrow\ k = \frac{3}{2} - \frac{1}{\sqrt{e}}$

23. Let $f : R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$. If $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim\limits_{x\to 2} g(x)$ is equal to :

(1) 18 (2) 36

(3) 24 (4) 12

Answer (1)

Sol. $\int\limits_6^{f(x)} 4t^3 dt = (x-2)g(x)$

$4(f(x))^3.f'(x) = g'(x)(x-2) + g(x)$

put $x = 2$,

$\frac{4(6)^3.1}{48} = g(2)$

$\lim\limits_{x\to 2} g(x) = 18$

24. The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in :

(1) Third and fourth quadrants only

(2) First, third and fourth quadrants

(3) First, second and fourth quadrants

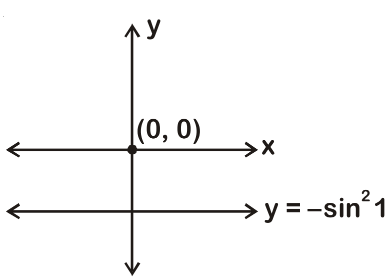
(4) Second and third quadrants only

Answer (1)

Sol. $y = \sin x \cdot \sin(x + 2) - \sin^2(x + 1)$

$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$

$= \frac{(\cos 2)-1}{2} = -\sin^2 1$



Graph of y lies in

III and IV Quadrant

25. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to :

(1) $-\frac{2}{3}$ (2) 6

(3) $\frac{10}{3}$ (4) $\frac{8}{3}$

Answer (2)

Sol. $y^2 = 4x$

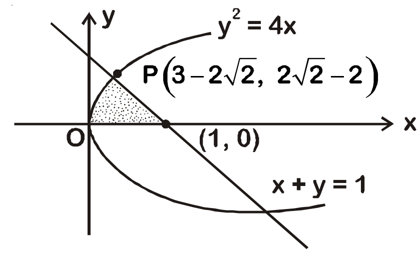
$x + y = 1$

$y^2 = 4(1 - y)$

$y^2 + 4y - 4 = 0$

$(y + 2)^2 = 8$

$y + 2 = \pm 2\sqrt{2}$



required area

$= \int\limits_0^{3-2\sqrt{2}} 2\sqrt{x}\, dx + \frac{1}{2}\times\left(2\sqrt{2}-2\right)\times\left(2\sqrt{2}-2\right)$

$= \left[2\times\frac{2}{3}x^{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}\left(8+4-8\sqrt{2}\right)$

$= \frac{4}{3}\times(3-2\sqrt{2})\sqrt{3-2\sqrt{2}} + 6 - 4\sqrt{2}$

$= \frac{4}{3}(3-2\sqrt{2})(\sqrt{2}-1) + 6 - 4\sqrt{2}$

$= \frac{4}{3}(3\sqrt{2}-3-4+2\sqrt{2}) + 6 - 4\sqrt{2}$

$= \left(6-\frac{28}{3}\right) + \left(\frac{20}{3}-4\right)\sqrt{2}$

$= -\frac{10}{3} + \frac{8}{3}\sqrt{2}$

$\Rightarrow\ a - b = \frac{10}{3} + \frac{8}{3} = 6$

26. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to :

(1) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$ (2) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$

(3) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$ (4) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

Answer (1)

Sol. $-\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{12}{13}\right) = -\sin^{-1}\left(\frac{3}{5}\times\frac{5}{13} - \frac{12}{13}\times\frac{4}{5}\right)$

$(\because xy \ge 0 \text{ and } x^2 + y^2 \le 1)$

$$= -\sin^{-1}\left(\frac{-33}{65}\right)$$

$$= \sin^{-1}\left(\frac{33}{65}\right)$$

$$= \cos^{-1}\left(\frac{56}{65}\right)$$

$$= \frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$$

27. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to :

(1) $\frac{1}{2}$ (2) 1

(3) –1 (4) $-\frac{1}{2}$

Answer (3)

Sol. $\int_0^{\pi/2} \frac{\cot x \, dx}{\cot x + \operatorname{cosec} x}$

$$= \int_0^{\pi/2} \frac{\cos x \, dx}{1+\cos x} = \int_0^{\pi/2} \left(1 - \frac{1}{1+\cos x}\right) dx$$

$$= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx$$

$$= \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$$

$$= \frac{\pi}{2} - \left[\tan\frac{x}{2}\right]_0^{\pi/2}$$

$$= \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right)$$

$m = \frac{1}{2}, n = -2$

$\Rightarrow$ mn = –1

28. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x)$ = ((hof)og)(x), then $\phi\left(\frac{\pi}{3}\right)$ is equal to :

(1) $\tan\frac{5\pi}{12}$ (2) $\tan\frac{\pi}{12}$

(3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{7\pi}{12}$

Answer (3)

Sol. $\phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right)$

$$= h\left(f\left(\sqrt{3}\right)\right) = h(3^{\frac{1}{4}})$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}} = -\frac{1}{2}(1+3-2\sqrt{3}) = \sqrt{3}-2 = -(-\sqrt{3}+2)$$

$$= -\tan 15° = \tan(180° - 15°) = \tan\left(\pi - \frac{\pi}{12}\right)$$

$$= \tan\frac{11\pi}{12}$$

29. The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents :

(1) The line through the origin with slope –1

(2) A circle of radius $\frac{1}{2}$

(3) A circle of radius 1

(4) The line through the origin with slope 1

Answer (4)

Sol. $|z - 1| = |z - i|$

Let $z = x + iy$

$(x - 1)^2 + y^2 = x^2 + (y - 1)^2$

$1 - 2x = 1 - 2y$

$\Rightarrow$ $x - y = 0$

Locus is straight line with slope 1

30. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(1) 3 (2) 5

(3) 4 (4) 7

Answer (2)

Sol. $1 + \sin^4 x = \cos^2 3x$

L.H.S = $1 + \sin^4 x$, R.H.S = $\cos^2 3x$

L.H.S ≥ 1 R.H.S ≤ 1

$\Rightarrow$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$ and $\cos^2 3x = 1$

$\sin x = 0$ and $(4\cos^2 x - 3)^2 \cos^2 x = 1$

$\Rightarrow$ $\sin x = 0$ and $\cos^2 x = 1$

$\Rightarrow$ $x = 0, \pm\pi, \pm 2\pi$

$\Rightarrow$ Total number of solutions is 5