CBSE Test Paper-01

Class - 12 Physics (Alternating current)

1. A 200 ohm resistor is connected in series with a $5\mu F$ capacitor. The voltage across the resistor is V_R = (1.20 V) cos(2500 rad/s)t.Capacitive reactance is
   a. 70 Ω
   b. 80 Ω
   c. 60 Ω
   d. 90 Ω
2. The current in a series LCR circuit will be maximum, then ω is:
   a. as large as possible
   b. $\sqrt{LC}$
   c. $\sqrt{LCR}$
   d. equal to natural frequency of LCR system
3. A series circuit consists of an ac source of variable frequency, a 115.0 Ω resistor, a 1.25 μF capacitor, and a 4.50-mH inductor. Impedance of this circuit when the angular frequency of the ac source is adjusted to twice the resonant angular frequency is
   a. 146.0 Ω
   b. 176.0 Ω
   c. 166.0 Ω
   d. 156.0 Ω
4. For high frequency capacitor offers:
   a. Less resistance
   b. More resistance
   c. None of these
   d. Zero resistance
5. Effective voltage V_{rms} is related to peak voltage V_o by
   a. V_{rms} = 0.707 V_o
   b. V_{rms} = 0.787V_o
   c. V_{rms} = 0.9 V_o

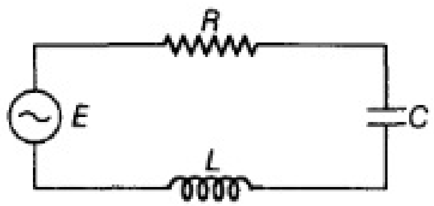
d. $V_{rms} = 0.5\ V_o$

6. An AC current, $I = I_0 \sin \omega t$ produces certain heat H in a resistor R over a time $T = 2\pi/\omega$. Write the value of the DC current that would produce the same heat in the same resistor in the same time.

7. Define the term rms value of the current. How is it related to the peak value?

8. Define the term wattless current.

9. The figure shows a series L-C-R circuit connected to a variable frequency 250 V source with L = 50 mH, C = 80μF and R = 40Ω .

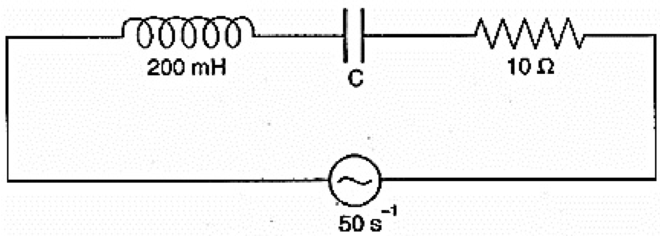


   i. the source frequency which drives the circuit in resonance.
   ii. the quality factor (Q) of the circuit.

10. The number of turns in secondary coil of a transformer is 100 times the number of turns in the primary coil. What is the transformation ratio?

11. In the following circuit, calculate,

   i. the capacitance 'c' of the capacitor if the power factor of the circuit is unity, and
   ii. also calculate the Q-factor of the circuit.



12. A source of AC voltage $V = V_0 \sin \omega t$ is connected to a series combination of a resistor 'R' and a capacitor 'C'. Draw the phasor diagram and use it to obtain the expression for

   i. impedance of the circuit and
   ii. phase angle.

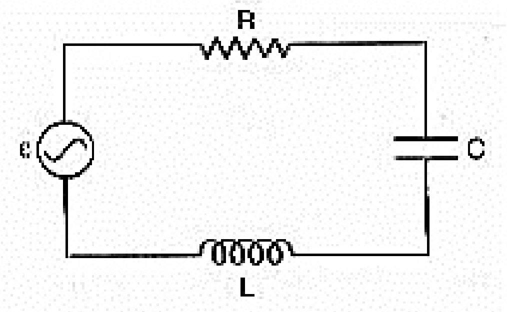
13. i. For a given AC, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2} i_m^2 R$.

ii. A light bulb is rated at 100 W for a 220V AC supply. Calculate the resistance of the bulb.

14. An LC circuit contains a 20 mH inductor and a $50 \mu F$ capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.

a. What is the total energy stored initially? Is it conserved during LC oscillations?
b. What is the natural frequency of the circuit?
c. At what time is the energy stored
   i. completely electrical (i.e. stored in the capacitor)?
   ii. completely magnetic (i.e. stored in the inductor)?
d. At what times is the total energy shared equally between the inductor and the capacitor?
e. If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

15. A series of LCR circuit connected to a variable frequency 230 V source, L = 5.0 H, $C = 80 \mu F, R = 40 \Omega$



a. Determine the source frequency which drives the circuit in resonance.
b. Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
c. Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

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Answers

1. b. 80Ω

   Explanation: V_R = (1.20 V) cos(2500 rad/s)t

   $\omega = 2500 rad/s$

   $C = 5\mu F = 5 \times 10^{-6} F$

   Capacitive reactance

   $X_C = \frac{1}{\omega C} = \frac{1}{2500 \times 5 \times 10^{-6}} = 80\Omega$

2. d. equal to natural frequency of LCR system

   Explanation: for maximum current in LCR series circuit impedance Z will be minimum

   $i = \frac{E}{Z}$

   $Z = \sqrt{R^2 + (X_L - X_C)^2}$

   impedance Z will be minimum when $X_L = X_C$

   hence

   $\omega L = \frac{1}{\omega C}$

   $\omega = \frac{1}{\sqrt{LC}}$

   this is equal to natural frequency of LCR system

3. a. 146.0

   Explanation: $R = 115\Omega$

   $C = 1.25\mu F = 1.25 \times 10^{-6} F$

   $L = 4.5mH = 4.5 \times 10^{-3} H$

   resonant angular frequency

   $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.5 \times 10^{-3} \times 1.25 \times 10^{-6}}} = \frac{1}{7.5 \times 10^{-5}}$

   given that the angular frequency of the ac source

   $\omega = 2\omega_0 = \frac{2}{7.5 \times 10^{-5}} = 26666.6 rad/s$

   impedance

   $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$= \sqrt{115^2 + \left[(26666.6 \times 4.5 \times 10^{-3}) - \left(\frac{1}{26666.6 \times 1.25 \times 10^{-6}}\right)\right]^2}$$
$$Z = 146\Omega$$

4. a. Less resistance

   Explanation: capacitive reactance

   $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$
   $$X_C \propto \frac{1}{C}$$

   hence, for high frequency capacitor offers less resistance.

5. a. V_{rms} = 0.707 V_0

   Explanation: Average value of V^2 over a complete cycle is given by

   $$\bar{V}^2 = \frac{1}{T}\int_0^T V^2 dt$$
   $$V = V_o \sin \omega t$$
   $$T = \frac{2\pi}{\omega}$$
   $$\bar{V}^2 = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} V_o{}^2 \sin^2 \omega t dt = \frac{\omega}{2\pi} V_o{}^2 \int_0^{2\pi/\omega} \frac{(1-\cos 2\omega t)}{2} dt$$
   $$\bar{V}^2 = \frac{\omega}{2\pi}\frac{V_o{}^2}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{2\pi/\omega} = \frac{\omega}{2\pi}\frac{V_o{}^2}{2}\left(\frac{2\pi}{\omega}\right)$$
   $$\bar{V}^2 = \frac{V_0^2}{2}$$

   The root-mean-square value of the alternating voltage is given by

   $$V_{rms} = \sqrt{\bar{V}^2} = \frac{V_o}{\sqrt{2}}$$

   V_{rms} = 0.707 V_0

6. Heat produced by DC is H = I^2RT ....(i)

   Heat produced by AC is

   $$H = I_V^2 RT \quad \text{or} \quad H = \left(\frac{I_0}{\sqrt{2}}\right)^2 RT \text{ ....(ii)}$$

   Where I_V = $I_0/\sqrt{2}$ = rms value of the AC current

   From Eqs. (i) and (ii), we get

   $$I^2 RT = \frac{I_0^2 RT}{2} \text{ or } I = I_0/\sqrt{2}$$

   where I stands for DC and I_0 is the peak value of AC current.

7. It is defined as the value of Alternating Current (AC) over a complete cycle which

would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle. It is also called virtual value or effective value of AC.

Let the peak value of the current be I_0

$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} \Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}}$

Where, I_0 peak value of AC.

8. The current in an AC circuit is said to be Wattless Current when the average power consumed in such circuit corresponds to Zero.Such current is also called as Idle Current.

9. Given, L = 50mH = $50 \times 10^{-3} H$

   $C = 80\mu F = 80 \times 10^{-6} F$

   $R = 40\Omega, V = 200V$

   i. In the L-C-R, the resonant angular frequency when $X_L = X_C$

      $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times80\times10^{-6}}} = 500\text{rad/s}$

      $\omega = 2\pi v \Rightarrow v = \frac{\omega}{2\pi}$

      $\Rightarrow \quad v = \frac{500}{2\pi} = \frac{250}{\pi} = 79.61 \approx 80\text{Hz}$

   ii. Quality factor, $Q = \frac{\omega_0 L}{R} = \frac{500\times50\times10^{-3}}{40} = 0.625$

10. Transformation ratio

    $\Rightarrow k = \frac{N_s}{N_p}$

    Since $N_s = 100 \times N_p$

    Thus $k = \frac{100N_p}{N_p} = 100$

11. i. Power factor, $\cos\phi = \frac{R}{Z}$ or Z = R [For power factor unity $\cos\theta = 1$]

       $\therefore X_C = X_L$ or $\frac{1}{2\pi f\ C} = 2\pi f\ L$

       or $C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\times9.87\times(50)^2\times200\times10^{-3}}$

       $= 5 \times 10^{-5} F$

       or $C = 50\mu F$

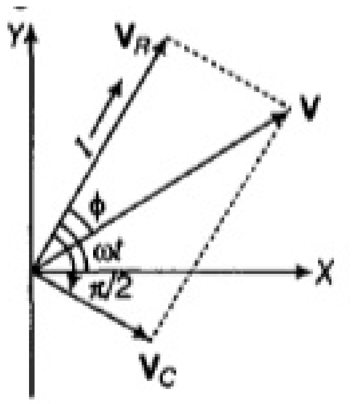
    ii. Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$

$Q = \frac{1}{10}\sqrt{\frac{200\times10^{-3}}{5\times10^{-5}}} = 6.32$

12. i. $V = V_0 \sin\omega t$

From diagram, by parallelogram law of vector addition, $V_R + V_C = V$

Using pythagorean theorem,



We get

$V^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 \Rightarrow V^2 = I^2\left(R^2 + X_C^2\right)$, X_C and R being the capacitive reactance and resistance of the resistor respectively.

$\therefore I = V/\sqrt{R^2 + X_C^2} = V/Z$ (say) where, Z = $\sqrt{R^2 + X_C^2} = \sqrt{R^2 + 1/\omega^2 C^2}$

Z = impedance of the circuit.

ii. The phase angle ϕ between resultant voltage and current is given by

$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega RC} \Rightarrow \phi = \tan^{-1}(\frac{1}{\omega RC})$

13. i. The average power dissipated,

$\overline{P} = \left(i^2 R\right) = \left(i_m^2 R \sin^2 \omega t\right) = i_m^2 R\left(\sin^2 \omega t\right)$

$\because \sin^2 \omega t = \frac{1}{2}(a - \cos 2\omega t)$

$\therefore \left(\sin^2 \omega t\right) = \frac{1}{2}[1 - (\cos 2\omega\theta)] = \frac{1}{2}(\because \cos 2\omega t = 0)$

$\therefore \quad \overline{P} = \frac{1}{2} i_m^2 R$

ii.) Power of the bulb P = 100 W

voltage, V = 220 V

$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\Omega$

14. a. Total initial energy

$E = \frac{Q_0^2}{2C} = \frac{10^{-2}\times10^{-2}}{2\times50\times10^{-6}} J$ = 1 J

This energy shall remain conserved in the absence of resistance.

b. Angular frequency, $\omega = \frac{1}{\sqrt{LC}}$

$= \frac{1}{(20\times10^{-3}\times50\times10^{-6})^{1/2}} Hz$

= 10^3 rads^{-1}

$v = \frac{10^3}{2\pi} Hz = 159 Hz$

c. $Q = Q_0 \cos \omega t$

Or $Q = Q_0 \cos \frac{2\pi}{T} t$, where $T = \frac{1}{v} = \frac{1}{159} s$ = 6.3 ms

Energy stored is completely electrical at t = 0, T/2, 3T/2 . . .

Electrical energy is zero i.e. energy stored is completely magnetic at

$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$

d. At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots \left[\because Q = Q_0 \cos \frac{\omega T}{8} = Q_0 \cos \frac{\pi}{4} = \frac{Q_0}{\sqrt{2}}\right]$

$\therefore$ Electrical energy $= \frac{Q^2}{2C} = \frac{1}{2}\frac{Q_0^2}{2C}$, which is half of the total energy.

e. R damps out the LC oscillations eventually. The whole of the initial energy 1.0 J is eventually dissipated as heat.

15. Here, L = 5.0 H, $R = 40\Omega$

$C = 80\mu F = 80 \times 10^{-6} F$

E_V = 230 volt

$E_0 = \sqrt{0} E_v = \sqrt{2} \times 230V$

a. Resonance angular frequency,

$\omega r = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{5\times80\times10^{-6}}} = \frac{1}{2\times10^{-7}}$ = 50 rad/sec = 50 rad/sec.

b. Impedance $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At resonance, $\omega L = \frac{1}{\omega C}$

$Z = \sqrt{R^2} = R = 40\Omega$

Amplitude of current at resonating frequency

$I_0 = \frac{E_0}{z} = \frac{\sqrt{2}\times230}{40}$ = 8.13 amp

$I_v = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} = 5.75 amp$

c. Potential drop across L

$V_{L\ rms} = I_v \omega_r L = 5.75 \times 50 \times 5.0$ = 1437.5 V

Potential drop across R

$V_{R\ rms} = I_v \times R = 5.75 \times 40$ = 230 volt

Potential drop across C

$V_{C\ rms} = I_v(\frac{1}{\omega_r C})$

$= 5.75 \times \frac{1}{50 \times 80 \times 10^-6}$

$= \frac{5.75}{4} \times 10^3$ = 1437.5 V

Potential drop across LC circuit

$V_{LC\ rms} = V_{L\ rms} - V_{C\ rms} = 0$